Math 2250-004

Week 10 concepts and homework, due Wednesday March 28 at the start of class

Recall that all listed problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

5.5: finding particular solutions using the method of undetermined coefficients; using variation of parameters; using the general solution $y = y_p + y_H$ to solve associated initial value problems.

5.5: 2, **3**, **10**, 21, 27, 29, **31**, 34.

w10.1) Consider the 3[nd] order differential operator for $y(x)$:

$$L(y) := y''' - 3y' + 2y.$$

a) Find the solution space to the homogeneous differential equation $L(y) = 0$. Hint: first find an integer root of the characteristic polynomial, then do long division.

b) Use the method of undetermined coefficients to find a particular solution to $L(y) = x$.

c) Use the method of undetermined coefficients to find a particular solution to $L(y) = e^{3x}$.

d) Use the method of undetermined coefficients to find a particular solution to $L(y) = e^{-2x}$. Hint: this will involve a lot of product rule differentiation if you just plug in the correct undetermined coefficients trial solution. An alternate shortcut you might consider would be to factor L so that $(D + 2)$ is one of the factors and is the part of L you apply first. We do an example like this in the notes that we cover Friday: Exercise 7, Wednesday March 22.

e) Use your work in a,b,c,d and linearity (superposition) to write down the general solution to

$$L(y) = 8x + 20e^{3x} - 8e^{-2x}.$$

f) Hand in a technology check (e.g. Wolfram alpha) for your answer to **e**.

5.5: non-standard right-hand sides.

5.5: **43**, 45

5.5: variation of parameters.

5.5: **52,** 57, 58.

w10.2 (Extra credit, 4 points) Find a particular solution to 5.5.52 using differential operator ideas, as follows. This is for the differential equation

$$y'' + 9y = \sin(3x).$$

a) Compute

$$L(x e^{3ix}) = [D^2 + 9](x e^{3ix}) = [D + 3i] \circ [D - 3i](x e^{3ix}).$$

Your answer should be

$$L(x e^{3ix}) = 6i e^{3ix}.$$

b) Then use Eulers formula and linearity to write $L(x e^{3ix})$ in terms of $L(x\cos(3x))$ and $L(x\sin(3x))$. By also expanding the right side of the identity above into real and imaginary functions you can deduce $L(x\cos(3x))$ and $L(x\sin(3x))$. Then you can deduce the particular solution you're looking for. This is the same solution as the undetermined coefficients method would give you, and differs from the one you found in problem 52 by a solution to the homogeneous DE.

5.6: solving forced oscillation problems; understanding beating and resonance in undamped problems, steady periodic and transient solutions in damped problems, and practical resonance in slightly damped problems; mass-spring applications; finding natural frequencies in more general (undamped) conservative systems via conservation of energy equations.

5.6: **1**, **7**, **11**, 3, 5, 7, 8, 11, 13, **17**, 18, **20**, 21, 22 .