Syllabus for Math 2250-004 Differential Equations and Linear Algebra

Spring 2017

Instructor Professor Nick Korevaar
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office hours T 4:30-6:00 p.m. LCB 204 (subject to change), and by appointment.
Lecture MTWF 10:45-11:35 a.m. MWF in WEB L105, T in JWB 335

Laboratory sections with Patrick Kilmer-Webb, webbp.math@gmail.com
2250-005 H 10:45-11:35 a.m. LCB 219
2250-006 H 9:40-10:30 a.m. AEB 310
with Kevin Childers, kevinrchilders@gmail.com
2250-015 H 10:45-11:35 a.m. JWB 308
2250-016 H 9:40-10:30 a.m. JWB 308

Course websites

Daily lecture notes and weekly homework assignments will be posted on our public home page.

http://www.math.utah.edu/~korevaar/2250spring17

Most students find that using and annotating the notes is helpful in understanding the class material. Some lecture material is included in the notes and there are also large open spaces in which we will work out examples together. My goal is to have weekly class notes posted on or by the preceding Friday. This should give ample time for you to print them out or to download electronic versions that you can annotate. Printing for math classes is free in the Math Department Rushing Student Center, in the basement of LCB. After class I will post filled-in versions of the notes but it will still be to your benefit to have attended and actively participated in the discussion and work leading to the filled-in versions. Class discussion will often be related to homework and lab problems.

later this week

Grades will be posted on our CANVAS course page; access via Campus Information Systems.

Textbook *Linear Algebra & Differential Equations with Introductory Partial Differential Equations and Fourier Series*, ISBN-13: 978-1-269-42557-5. • I'll check what's up & post Chapter 1

This text is a hybrid of the three texts: Differential Equations and Linear Algebra 3rd Edition, by Edwards and Penney; Applied Partial Differential Equations with Fourier Series and Boundary Value Problems, 5th edition, by Haberman; Elementary Linear Algebra, by Edwards and Penney. You should definitely buy this version of the text if you plan to take the 4th semester in the new engineering math sequence, Math 3140, or the PDE course Math 3150. If your math courses will terminate with Math 2250, then the 3rd edition Differential Equations and Linear Algebra text by Edwards-Penney will suffice. (This was the text for Math 2250 thru summer 2013.)

Final Exam logistics: Thursday April 27, 10:30 a.m.-12:30 p.m., in our MWF classroom WEB L105. This is the University scheduled time and location.

Catalog description for Math 2250: This is a hybrid course which teaches the allied subjects of linear algebra and differential equations. These topics underpin the mathematics required for most students in the Colleges of Science, Engineering, Mines & Earth Science.

Prerequisites: Math 1210-1220 or 1310-1320 (or 1250-1260 or 1311-1321, i.e. single-variable calculus.) You are expected to have learned about vectors and parametric curves in one of these courses, or in Math 2210 or or Physics 2210 or 3210. Practically speaking, you are better prepared for this course if you've had elements of multivariable calculus in courses such as 1320, 1321, or 2210 and if your grades in the prerequisite courses were above the "C" level.

Learning Objectives for 2250

The goal of Math 2250 is to master the basic tools and problem solving techniques important in differential equations and linear algebra. These basic tools and problem solving skills are described below.

The essential topics

Be able to model dynamical systems that arise in science and engineering, by using general principles to derive the governing differential equations or systems of differential equations. These principles include linearization, compartmental analysis, Newton's laws, conservation of energy and Kirchoff's law.

Learn solution techniques for first order separable and linear differential equations. Solve initial value problems in these cases, with applications to problems in science and engineering. Understand how to approximate solutions even when exact formulas do not exist. Visualize solution graphs and numerical approximations to initial value problems via slope fields. Understand phase diagram analysis for autonomous first order differential equations.

Become fluent in matrix algebra techniques, in order to be able to compute the solution space to linear systems and understand its structure; by hand for small problems and with technology for large problems.

read these!

Be able to use the basic concepts of linear algebra such as linear combinations, span, independence, basis and dimension, to understand the solution space to linear equations, linear differential equations, and linear systems of differential equations.

Understand the natural initial value problems for first order systems of differential equations, and how they encompass the natural initial value problems for higher order differential equations and general systems of differential equations.

Learn how to solve constant coefficient linear differential equations via superposition, particular solutions, and homogeneous solutions found via characteristic equation analysis. Apply these techniques to understand the solutions to the basic unforced and forced mechanical and electrical oscillation problems.

Learn how to use Laplace transform techniques to solve linear differential equations, with an emphasis on the initial value problems of mechanical systems, electrical circuits, and related problems.

Be able to find eigenvalues and eigenvectors for square matrices. Apply these matrix algebra concepts to find the general solution space to first and second order constant coefficient homogeneous linear systems of differential equations, especially those arising from compartmental analysis and mechanical systems.

Understand and be able to use linearization as a technique to understand the behavior of nonlinear dynamical systems near equilibrium solutions. Apply these techniques to non-linear mechanical oscillation problems. (Additional material, subject to time availability: Apply linearization to autonomous systems of two first order differential equations, including interacting populations. Relate the phase portraits of non-linear systems near equilibria to the linearized data, in particular to understand stability.)

Develop your ability to communicate modeling and mathematical explanations and solutions, using technology and software such as Maple, Matlab or internet-based tools as appropriate.

Problem solving fluency

Students will be able to read and understand problem descriptions, then be able to formulate equations modeling the problem usually by applying geometric or physical principles. Solving a problem often requires specific solution methods listed above. Students will be able to select the appropriate operations, execute them accurately, and interpret the results using numerical and graphical computational aids.

Students will also gain experience with problem solving in groups. Students should be able to effectively transform problem objectives into appropriate problem solving methods through collaborative discussion. Students will also learn how to articulate questions effectively with both the instructor and TA, and be able to effectively convey how problem solutions meet the problem objectives.

Week-by-Week Topics Plan

Topic schedule is subject to slight modifications as the course progresses, but exam dates are fixed.

Week 1: 1.1-1.4; differential equations, mathematical models, integral as general and particular solutions, slope fields, separable differential equations.

Week 2: 1.4-1.5, EP 3.7, 2.1-2.2; separable equations cont., linear differential equations, circuits, mixture models, population models,equilibrium solutions and stability.

Week 3: 2.2-2.4; equilibrium solutions and stability cont., acceleration-velocity models, numerical solutions.

Week 4: 2.5-2.6, 3.1; numerical solutions cont., linear systems; Super quiz over chapters 1-2.

Week 5: 3.1-3.4; linear systems, matrices, Gaussian elimination, reduced row echelon form, matrix operations.

Week 6: 3.5-3.6; matrix inverses, determinants, review; **Midterm exam 1 on Friday February 17** covering material from weeks 1-6.

Week 7: 4.1-4.3; vector spaces, linear combinations in $\mathbb{R}^n$, span and independence, subspaces, bases and dimension.

Week 8: 4.4, 5.1-5.3; second-order linear DEs, general solutions, superposition, homogeneity and constant coefficients.

Week 9: 5.3-5.5; mechanical vibrations, pendulum model, particular solutions to non-homogeneous problems.

Week 10: 5.5-5.6, EP 3.7; forced oscillations and associated physical phenomena.practical resonance Laplace transforms, solving IVPs with transforms, partial fractions and translations.

Week 11: 10.1-3; Laplace transforms, solving IVPs with transforms, partial fractions and translations. **Midterm exam 2 on Friday March 31** covering material from weeks 7-11.

out of text order

Week 12: 10.4-10.5, EP 7.6, 6.1-6.2 Unit steps, convolutions, impulse function forcing; eigenvalues, eigenvectors and diagonalizability.

Week 13: 6.1-6.2 continued; 7.1-7.3; first order systems of differential equations; framework for differential equations in which every DE is equivalent to a first order system of DE's. Matrix systems of DEs

Week 14: 7.3-7.4; solution algorithms and applications for first and second order systems of differential equations; input-output modeling and mechanical systems.

Week 15: 7.3-7.4 continued (or 9.1-9.3 if time is available), and review. **Final exam Thursday April 27, 10:30 a.m. - 12:30 p.m. in classroom WEB L105. This is the University scheduled time.**

Prof. Korevaar

Math 2250-004 Week 1 notes
We will not necessarily finish the material from a given day's notes on that day. We may also add or subtract some material as the week progresses, but these notes represent an in-depth outline of what we will cover. These notes are for sections 1.1-1.3, and part of 1.4.

Monday January 9

• Go over course information on syllabus and course homepage:

http://www.math.utah.edu/~korevaar/2250spring17

§1.1 & 1.2

• Note that there is a quiz this Wednesday on the material we cover today and tomorrow, and that your first lab meeting is this Thursday. Your first homework assignment will be due next Wednesday, January 17.

Then, let's begin!

• What is an n^{th} order differential equation (DE)?

any equation involving a function $y = y(x)$ and its derivatives, for which the highest derivative appearing in the equation is the n^{th} one, $y^{(n)}(x)$; i.e. any equation which can be written as

$$F\big(x, y(x), y'(x), y''(x), ... y^{(n)}(x)\big) = 0.$$

Exercise 1: Which of the following are differential equations? For each DE determine the order.

a) For $y = y(x)$, $(y''(x))^2 + \sin(y(x)) = 0$ — yes! 2nd order

b) For $x = x(t)$, $x'(t) = 3\,x(t)(10 - x(t))$. — yes! 1st order

c) For $x = x(t)$, $x' = 3\,x\,(10 - x)$. — yes! same as (b)! abbreviated (don't get fooled)

d) For $z = z(r)$, $z'''(r) + 4\,z(r)$. — No! no "=" sign! not an equation

$z'''(r) + 4z(r) = \sin(r) + 7$ is a DE (3rd order)

e) For $y = y(x)$, $y' = y^2$. — YES. order 1st

A solution function $y(x)$ to the differential equation $F\left(x, y, y', y'', y^{(n)}\right) = 0$ defined on some interval I is any function $y(x)$ which makes the differential equation a true equality for all x in I.

sol's are fcns, not numbers!

A solution function $y(x)$ to a first order differential equation $F(x, y, y') = 0$ on the interval I which also satisifies $y\left(x_0\right) = y_0$ for a specified $x_0 \in I$ and $y_0 \in \mathbb{R}$ is called a solution to the initial value problem (IVP)

$$\text{IVP} \quad \begin{cases} F(x, y, y') = 0 \\ y\left(x_0\right) = y_0 \end{cases}$$

· Initial condition

Exercise 2: Consider the differential equation $\dfrac{dy}{dx} = y^2$ from (1e).

2a) Show that functions $y(x) = \dfrac{1}{C - x}$ solve the DE (on any interval not containing the constant C).

2b) Find the appropriate value of C to solve the initial value problem

$$y' = y^2$$
$$y(1) = 2 .$$

LHS RHS

2a) $\dfrac{dy}{dx} = (y(x))^2$

are $y = \dfrac{1}{C-x}$ sol'n's (C is any constant)?

plug into DE, & see if we get a true identity.

$y = (C-x)^{-1}$

LHS $y' = -1(C-x)^{-2}(-1) = (C-x)^{-2} = \dfrac{1}{(C-x)^2}$

RHS $y^2 = \left(\dfrac{1}{C-x}\right)^2$

$y' = y^2$? yes

LHS = RHS for these fcns $y(x)$, so they are solutions to the DE.

2b) Solve IVP:

$y = \dfrac{1}{C-x}$

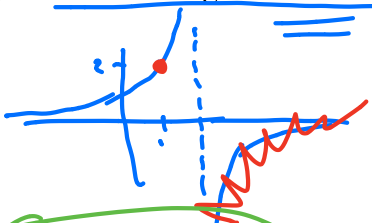
$y(1) = 2$

$y(1) = \dfrac{1}{C-1} = 2 \Rightarrow C - 1 = \frac{1}{2} \Rightarrow C = 3/2$

soln $\boxed{y(x) = \dfrac{1}{3/2 - x}}$

$y(1) = \dfrac{1}{3/2 - 1} = \dfrac{1}{1/2} = 2$ ✓

2c) What is the largest interval on which your solution to (b) is defined as a differentiable function? Why?



$y(x) = \frac{1}{3/2 - x}$

$y(1) = 2$

interval to contain $x_0 = 1$

$-\infty < x < 3/2$

2d) Do you expect that there are any other solutions to the IVP in 2b? Hint: The graph of the IVP solution function we found is superimposed onto a "slope field" below: The line segments at points (x, y) have values y^2, because solutions graphs to the differential equation

$$\boxed{y' = y^2}$$

will have slopes given by the derivatives of the solutions $y(x)$. This might give you some intuition about whether you expect more than one solution to the IVP.

the graph of any soln $y = y(x)$,

(1) has slope y' at any pt (x, y) on graph

(2) but because $y(x)$ solves the DE, the slope is also y^2

No: (I misread the question in class)

~~Yes~~
- know where we start
- know slope where-ever we are
- should only be one graph

Expect unique solutions

$y' = y^2$

$y = \frac{1}{3/2 - x}$

$y(1) = 2$

slopes 9 along $y = 3$

$y = 2$ slopes 4

$y = 1$ slopes $y^2 = 1$

green slopes $= 0$ along $y = 0$