Exercise 1 (today): Here's another example of using a separable DE to illustrate the existence-uniqueness theorem.

a) Does each IVP

$$y' = x^2 y^2$$
$$y(x_0) = y_0$$

have a unique solution?

$$\left\{ \begin{array}{ll} f(x,y) = x^2 y^2 & \text{cont on } \mathbb{R}^2 \\ \frac{\partial f}{\partial y} = x^2 2y & \text{cont on } \mathbb{R}^2 \end{array} \right.$$

So sol'ns to IVP's exist & are unique!

b) Find all solutions to this differential equation.

And illustrate dfield.

Wednesday starts here

$$\frac{dy}{dx} = x^2 y^2$$

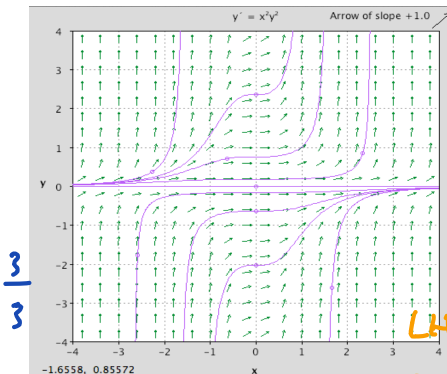
$$\int \frac{dy}{y^2} = \int x^2\,dx \quad \text{if } y \neq 0$$

$$\frac{1}{y^2} = y^{-2}, \quad \int y^{-2}\,dy = \frac{y^{-1}}{-1} = \frac{x^3}{3} + C$$

$$-\frac{1}{y} = \frac{x^3}{3} + C$$

$$\Rightarrow -y = \frac{1}{\frac{x^3}{3} + C}$$

$$\Rightarrow y = \frac{-1}{\frac{x^3}{3} + C} \cdot \frac{3}{3}$$



if $y = 0$

$y(x) = 0$ is also a sol'n - then

LHS $y'(x) = 0$

RHS $x^2 y^2 = 0$

$$\left\{ \begin{array}{l} y' = x^2 y^2 \\ y(0) = 0 \end{array} \right.$$

Maple check (notice it misses the singular solution):

> with(DEtools):

dsolve(y'(x) = x^2·y(x)^2, y(x));

$$y(x) = \frac{3}{-x^3 + 3\_C1} \qquad (1)$$

$$\boxed{y = \frac{-3}{x^3 + 3C}} \text{ "="} \frac{-3}{x^3 + C}$$

//

won't work

sol'n: $y(x) = 0$

>

Exercise 2: Do the initial value problems below always have unique solutions? Can you find them? (Notice these are NOT separable differential equations.) Can Maple find formulas for the solution functions?

a)

$$y' = x^2 + y^2$$
$$y(x_0) = y_0$$

slope function $f(x,y) = x^2 + y^2$ is cont on $\mathbb{R}^2$ (entire x-y plane)

$\frac{\partial f}{\partial y} = 2y$ also cont on $\mathbb{R}^2$

"there exists" ↳ $\exists!$ sol'ns to each IVP.

"unique"

b)

$$y' = x^4 + y^4$$
$$y(x_0) = y_0$$

$$\left. \begin{array}{l} f = x^4 + y^4 \\ \frac{\partial f}{\partial y} = 4y^3 \end{array} \right\} \text{continuous on } \mathbb{R}^2$$

In Maple, we saw that even though sol'ns exist, there might not be a formula for them

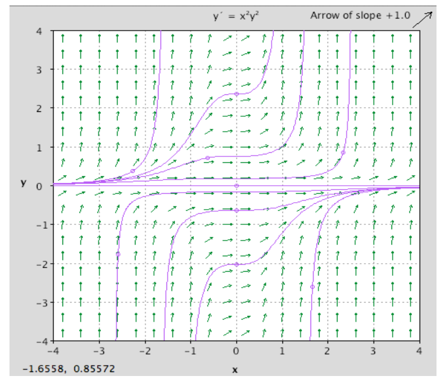
Exercise 1 (today): Here's another example of using a separable DE to illustrate the existence-uniqueness theorem.

a) Does each IVP

$$y' = x^2 y^2$$
$$y(x_0) = y_0$$

have a unique solution?

b) Find all solutions to this differential equation.



Maple check (notice it misses the singular solution):

> $with(DEtools)$:
> $dsolve\left(y'(x) = x^2 \cdot y(x)^2, y(x)\right);$

$$y(x) = \frac{3}{-x^3 + 3\_C1} \qquad \textbf{(1)}$$

>

Exercise 2: Do the initial value problems below always have unique solutions? Can you find them? (Notice these are NOT separable differential equations.) Can Maple find formulas for the solution functions?

a)

$$y' = x^2 + y^2$$
$$y(x_0) = y_0$$

> $with(DEtools);$

> $dsolve\left(y'(x) = x^2 + y(x)^2, y(x)\right);$

$$y(x) = \frac{\left(-\mathrm{BesselJ}\left(-\frac{3}{4}, \frac{1}{2} x^2\right) \_C1 - \mathrm{BesselY}\left(-\frac{3}{4}, \frac{1}{2} x^2\right)\right) x}{\_C1\, \mathrm{BesselJ}\left(\frac{1}{4}, \frac{1}{2} x^2\right) + \mathrm{BesselY}\left(\frac{1}{4}, \frac{1}{2} x^2\right)} \qquad \textbf{(2)}$$

>

b)

$$y' = x^4 + y^4$$

$$y(x_0) = y_0$$

```
> dsolve(y'(x) = x^4 + y(x)^4, y(x));
>
>
```

Math 2250-004
Wed Jan 18 Quiz at end of class!
1.4: separable DEs, examples and experiment.

- Lab 2 tomorrow
- Quiz today
- Office hours today 4:30-6:00 LCB 218
- little bit of Tuesday
- bring laptops to lab

For your section 1.4 hw this week I assigned a selection of separable DE's - some applications will be familiar with from last week, e.g. exponential growth/decay and Newton's Law of cooling. Below is an application that might be new to you, and that illustrates conservation of energy as a tool for modeling differential equations in physics.

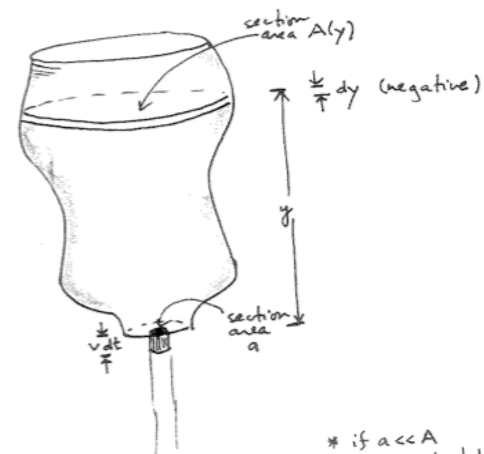
Toricelli's Law, for draining water tanks. Refer to the figure below.
Exercise 1:
a) Neglect friction, use conservation of energy, and assume the water still in the tank is moving with negligable velocity ($a << A$). Equate the lost potential energy from the top in time dt to the gained kinetic energy in the water streaming out of the hole in the tank to deduce that the speed v with which the water exits the tank is given by

$$v = \sqrt{2gy}$$

when the water depth above the hole is $y(t)$ (and g is accel of gravity).

b) Use part (a) to derive the separable DE for water depth

$$A(y)\frac{dy}{dt} = -k\sqrt{y} \quad (k = a\sqrt{2g}) .$$



* if a<<A we may neglect the KE of the slow moving water inside the vessel.