

Superquiz 3 Review Sheet
 Math 2250-010
 April 17, 2014 (for quiz on April 18)

Friday's superquiz covers material from 10.4-10.5, EP 7.6 supplement (Laplace transforms); 6.1-6.2 (eigenvalues, eigenvectors, eigenspaces, matrix diagonalizability, including complex eigenvalues and eigenvectors), and 7.1-7.4 (systems of differential equations). No technology beyond a simple scientific calculator is allowed. Graphing calculators, cell phones, etc. are not allowed. Symbolic answers are allowed so no calculator is really needed.

Chapter 10.4-10.5, EP 7.6, Laplace transform techniques.

Be able to use the Laplace transform table entries we've covered since the second midterm, especially in the context of solving differential equation initial value problems. (And don't forget the Laplace transform material from before the second midterm; table will be provided.) The new tools we developed have to do with these table entries:

$f(t)$ with $ f(t) \leq Ce^{Mt}$	$F(s) := \int_0^{\infty} f(t)e^{-st} dt$ for $s > M$	comments
$u(t - a)$ unit step function	$\frac{e^{-as}}{s}$	for turning components on and off at $t = a$.
$f(t - a)u(t - a)$	$e^{-as}F(s)$	more complicated on/off
$\delta(t - a)$	e^{-as}	unit impulse/delta "function"
$\int_0^t f(\tau)g(t - \tau) d\tau$	$F(s)G(s)$	convolution integrals to invert Laplace transform products

Chapter 6.1-6.2, eigenvectors, eigenvalues, eigenspaces, diagonalizable matrices.

Understand and be able to use the algorithm for finding eigenvalues (i.e. roots of the characteristic polynomial); for any eigenvalue be able to find an eigenspace basis; be able to determine whether or not a matrix is diagonalizable. Eigenvalue-eigenvector problems are most likely to arise on the superquiz in the context of a step towards solving a first order system of differential equations.

Chapter 7.1-7.4, systems of differential equations.

Understand how/why any differential equation IVP is equivalent to a first order system of differential equations IVP. Be able to go between these equivalent formulations in that case. (Most first order systems of differential equations do not arise from a higher order differential equations, however.)

Be able to find the solution space to homogeneous linear first order systems of DE's, by finding bases of the form $e^{\lambda t} \underline{y}$ when the matrix A in the system is constant and is diagonalizable with real eigenvectors.

Also know how to work with complex eigendata and complex solutions $e^{\lambda t} \underline{y}$ in order to find real-valued solutions. Be able to solve IVP's for homogeneous linear first order systems of differential equations.

Know how to model input-output systems, to arrive either at homogeneous or inhomogeneous linear systems of DE's (depending on whether solute is flowing into the system or not). Know how to solve the resulting systems of DE's, especially in the homogeneous case, but potentially using $\underline{x} = \underline{x}_p + \underline{x}_H$ in the inhomogeneous case.

Know how to model second order mass-spring systems, to arrive either at homogeneous or inhomogeneous linear systems of DE's (depending on whether there is external forcing or not). Know how to solve the resulting systems of DE's, especially in the homogeneous case but potentially using $\underline{x} = \underline{x}_p + \underline{x}_H$ in the inhomogeneous case with sinusoidal forcing.