Superquiz 2 Review Sheet
Math 2250-010
March 7, 2014

Friday's superquiz covers 4.1-4.4, 5.1-5.4, including this week's homework. No technology beyond a simple scientific calculator is allowed. Graphing calculators, cell phones, etc. are not allowed. Symbolic answers are allowed so no calculator is really needed, although a scientific calculator could give you confidence on an amplitude-phase calculation, for example.

Chapter 4.1-4.4
Memorize the key definitions:

vector space

linear combination of a collection $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \dots \underline{\boldsymbol{v}}_k$ of k vectors

linearly independent vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \dots \underline{\boldsymbol{v}}_k$

linearly dependent vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \dots \underline{\boldsymbol{v}}_k$

span of vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \dots \underline{\boldsymbol{v}}_k$

subspace (i.e. sub vector space) of a vector space

basis of a vector space

how do you get a basis if you already have a (possibly dependent) spanning set?

how do you get a basis if you have an independent set that doesn't span the vector space?

dimension of a vector space

Subspace examples from Chapter 4, involving the concepts above:
solution set to homogeneous matrix equation $[A]\underline{\boldsymbol{x}} = \underline{\boldsymbol{0}}$ (implicit description)
span of vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \dots \underline{\boldsymbol{v}}_k$ (explicit description)

Subspace examples from Chapter 5
solution set to homogeneous linear differential equation for e.g. $y = y(x)$ on an interval I, i.e. solutions to

$$L(y) := y^{(n)} + p_{n-1}(x)\, y^{(n-1)} + \dots + p_1(x) y' + p_0(x) y = 0$$

(implicit description)
span of functions $y_1, y_2, \dots y_k$ (explicit description)

What does it mean for a transformation $L : V \rightarrow W$ between vector spaces to be <u>linear</u>? Hints: we use these two properties to show that the homogeneous solution space for $L(y) = 0$ is a subspace, and to answer the next question below. These properties are also sometimes called the "**principle of superposition**".
ans: It must always be true that

$$L(y_1 + y_2) = L(y_1) + L(y_2)$$
$$L(c\,y) = c\,L(y), \text{ for } c \in \mathbb{R}.$$

What is the general solution to $L(y) = f$, if L is a linear transformation (or "operator"), in terms of particular and homogeneous solutions?
ans: $y = y_P + y_H$ *Do you know what that formula represents, and can you explain why it's true?*

Examples?
solution space to $[A]\underline{x} = \underline{b}$ where A is a matrix
solution space to $L(y) = f$, i.e. the non-homogeneous linear DE where

$$L(y) := a_n\,y^{(n)} + a_{n-1}\,y^{(n-1)} + \ldots + a_1\,y' + a_0\,y$$

What is the **natural initial value problem** for n^{th}-order linear differential equation, i.e. the one that has unique solutions? i.e. what initial conditions can you add below and get unique solutions?

$$L(y) := y^{(n)} + p_{n-1}(x)y^{(n-1)} + \ldots + p_1(x)y' + p_0(x)y = f(x)$$

ans: you get to specify $y(x_0), y'(x_0), y''(x_0) \ldots y^{(n-1)}(x_0)$.

What is the **dimension of the solution space to the n^{th} order homogeneous linear DE**

$$L(y) := y^{(n)} + p_{n-1}(x)y^{(n-1)} + \ldots + p_1(x)y' + p_0(x)y = 0 \ ?$$

ans: n

In what ways can you tell if functions $y_1(x), y_2(x), \ldots y_n(x)$ are a **basis** for the homogeneous solution space above?

How is your answer above related to a **Wronskian matrix** and the **Wronskian determinant**?

How do you find the **general solution** to the **homogeneous constant coefficient linear DE**

$$L(y) := a_n\,y^{(n)} + a_{n-1}\,y^{(n-1)} + \ldots + a_1\,y' + a_0\,y = 0\,?$$

(Your answer should involve the characteristic polynomial, and the various cases for creating solutions depending on whether roots are real, repeated, and/or complex. Do you remember Euler's formula, and can you use it? Do you remember the Taylor-Maclaurin series formula in general? For e^x, $\cos(x)$, $\sin(x)$ in particular?)

5.4 Mechanical vibrations

What are the governing second order DE's for a possibly **damped mass-spring configuration** (Newton's second law, linearized) ?

What are **unforced undamped oscillations**, and their **solution formulas/behavior**?
Can you convert a linear combination $A\cos(\omega t) + B\sin(\omega t)$ in amplitude-phase form $C\cos(\omega t - \alpha)$? Do you remember the addition angle formulas? Can you explain the physical properties of the solution?

What are the three kinds of damped behavior that arise in **unforced damped mass-spring configurations** (depending on the size of damping coefficient)? Can you identify them in problems when you have numerical values for m, c, k?

Can you solve and interpret IVP's for the undamped and damped configurations above?