Name________________________
Student I.D.___________________

Math 2250-010
Super Quiz 2
March 7, 2014

1a) What two properties must hold for a subset W of a vector space V in order that W be a <u>subspace</u>?

(1 point)

1b) What does it mean for a collection vectors $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$ to be a <u>basis</u> for a vector space/subspace W?

(1 point)

2a) The span of the three vectors

$$\underline{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 3 \\ 2 \end{bmatrix}, \ \underline{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix}, \ \underline{v}_3 = \begin{bmatrix} 4 \\ 1 \\ 12 \\ 1 \end{bmatrix}$$

is a subspace of $\mathbb{R}^4$. Find a basis for this subspace using some or all of the three vectors, and explain your reasoning. Hint: the following matrix, which has the three vectors as its columns, along with its reduced row echelon form on the right, can help you answer this question:

$$\begin{bmatrix} 2 & 0 & 4 \\ -1 & 1 & 1 \\ 3 & 2 & 12 \\ 2 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(5 points)

2b) What is the dimension of the subspace in part 2a?

(1 point)

3) Consider the following differential equation for a function $x(t)$, which could arise from an unforced mass-spring configuration.

$$x''(t) + 2x'(t) + 5x(t) = 0.$$

3a) Find the general solution to this homogeneous linear differential equation.

(6 points)

3b) Which of the three damping phenomena is exhibited by solutions to this differential equation?

(1 point)

3c) Now consider the inhomogeneous DE

$$x''(t) + 2x'(t) + 5x(t) = 20.$$

Notice that $x_p(t) = 4$ is a particular solution. Use this fact and your previous work to write down the general solution to the inhomogeneous DE.

(2 points)

3d) Solve the initial value problem

$$\begin{gathered} x''(t) + 2x'(t) + 5x(t) = 20 \\ x(0) = 0 \\ x'(0) = 0. \end{gathered}$$

(5 points)

4) Find the amplitude-phase form $x(t) = C\cos(3t - \alpha)$, for the function

$$x(t) = -\cos(3t) + \sqrt{3}\sin(3t).$$

(3 points)