Name________________________

Student I.D.____________________

Math 2250-010

Super Quiz 1

January 31, 2014

1a) Solve this initial value problem for a function $y(x)$:

$$y'(x) = \frac{-x}{y+1}$$
$$y(0) = 2$$

(6 points)

1b) Show that the initial value problem above arises from the following description of the function $y(x)$: The graph of the function $y(x)$ has the property that it goes through the point $(0, 2)$. Furthermore, every normal line to the graph (i.e. every line perpendicular to the graph) passes through the point $(0, -1)$. Hint: perpendicular lines have slopes m_1, m_2 hat are negative reciprocals, $m_1 m_2 = -1$. Thus you can relate the slopes of these normal lines through points (x, y) on the graph to the slopes of the graph at those points.

(2 points)

2) Consider the following linear drag initial value problem for a velocity function $v(t)$:

$$v'(t) = 30 - 2v$$
$$v(0) = 5.$$

a) Construct and use a phase diagram to determine $\lim_{t \to \infty} v(t)$ for the solution to this IVP.

(2 points)

b) Solve the initial value problem for $v(t)$, using the method for linear differential equations.

(6 points)

c) If the position function $x(t)$ corresponding to this velocity function satisfies $x(0) = 0$, find $x(t)$.

(4 points)

3) Consider a brine tank which initially contains 100 gallons of water, with concentration 0.1 pounds of salt per gallon. At time $t = 0$ hours water begins to flow into the tank at a rate of 50 gallons per hour, with concentration 0.8 pounds of salt per gallon. At the same time, the well-mixed brine solution begins to flow out of the tank at a rate of 60 gallons per hour, until the tank empties. Let $x(t)$ be the amount of salt in the tank at time t, until the tank becomes empty.

a) At what time will the tank become empty?

(1 points)

b) Find the initial value problem satisfied by the salt amount $x(t)$. You do not need to find the solution function $x(t)$ (because there is not so much time available on this superquiz).

(4 points)