Math 2250-010
Quiz 8
April 4, 2014
SOLUTIONS

1) Consider the matrix

$$A := \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}.$$

1a) Find the eigenvalues and eigenvectors (eigenspace bases).

(9 points)

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 1 \\ 4 & -1-\lambda \end{vmatrix} = (\lambda - 2)(\lambda + 1) - 4$$
$$= \lambda^2 - \lambda - 6 = (\lambda - 3)(\lambda + 2)$$

so the eigenvalues are $\lambda = 3$, $\lambda = -2$.

$E_{\lambda=3}$: We want to solve $(A - 3I)\underline{\mathbf{v}} = \underline{\mathbf{0}}$ which has augmented matrix

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 4 & -4 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Thus $v = [1, 1]^T$ is an eigenvector (eigenspace basis). (This is because $1 \cdot col_1 + 1 \cdot col_2 = \underline{\mathbf{0}}$. You'd end up at the same place if you used the Chapter 3 algorithm of back solving from reduced row echelon form, it would just take you longer:

$$v_2 = t,\ v_1 = t \Rightarrow \underline{\mathbf{v}} = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}.)$$

$E_{\lambda=-2}$: We want to solve $(A + 2I)\underline{\mathbf{v}} = \underline{\mathbf{0}}$ which has augmented matrix

$$\left[\begin{array}{cc|c} 4 & 1 & 0 \\ 4 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 4 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

Thus $v = [-1, 4]^T$ is an eigenvector (eigenspace basis). (This is because $-1 \cdot col_1 + 4 \cdot col_2 = \underline{\mathbf{0}}$.)

1b) Give a reason to explain why the two vectors you found in part (a) form a basis for $\mathbb{R}^2$.

(1 point)

The eigenvectors

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

are a basis for $\mathbb{R}^2$ because they are linearly independent and span $\mathbb{R}^2$. We know this because the matrix which has these vectors as columns,

$$\begin{bmatrix} 1 & -1 \\ 1 & 4 \end{bmatrix}$$

has non-zero determinant, equivalently reduces to the identity matrix.