Name________________________
Student I.D.___________________

Math 2250-010
Quiz 7 Take-home
March 21, 2014
Due at the start of class on Monday
SOLUTIONS

1) Consider the following four mechanical oscillation differential equations. In each case answer the following questions:

(i) Find the general solution, if the differential equation is homogeneous. If the problem is inhomogeneous write the "undetermined coefficients" guess for a particular solution. You do not need to find the numerical values of the undetermined coefficients on this quiz, although that is something you should be able to do.
(ii) What physical phenomenon is exhibited by the general solutions to this differential equation?

1a) $x''(t) + 2\,x'(t) + 17\,x(t) = 0$

(2 points)

(i)

$$p(r) = r^2 + 2\,r + 17 = (r+1)^2 + 16 = (r + 1 + 4\,i)(r + 1 - 4\,i)$$

has roots $r = -1 \pm 4\,i$ *so the homogeneous solution is*

$$x(t) = c_1 e^{-t} \cos(4\,t) + c_2 e^{-t} \sin(4\,t)$$

(ii) solutions exhibit underdamping.

1b) $x''(t) + 16\,x(t) = 0$

(2 points)

(i)

$$p(r) = r^2 + 16 = (r + 4\,i)(r - 4\,i)$$

has roots $r = \pm 4\,i$ *so the general homogeneous solution is*

$$x(t) = A\cos(4\,t) + B\sin(4\,t) = C\cos(4\,t - \alpha)$$

(ii) solutions exhibit undamped, unforced, simple harmonic motion.

1c) $x''(t) + 16\,x(t) = 4\cos(4\,t)$.

(3 points)

(i)

$$p(r) = r^2 + 16 = (r + 4\,i)(r - 4\,i)$$

since the forcing function solves the homogeneous DE, (because it arises from the exponentials $e^{\pm 4\,i\,t}$ *that arise from the roots of the characteristic polynomial, we must multiply the undetermined coefficents guess by* t^1*, ie.*

$$x_P(t) = t\cdot(A\cos(4\,t) + B\sin(4\,t)).$$

(In fact, it turns out that $x_P(t) = B\,t\sin(4\,t)$ *.)*

(ii) solutions exhibit pure resonance.

1d) $x''(t) + 16\,x(t) = 4\cos(4.5\,t)$.

(i) Since the forcing angular frequency $\omega = 4.5 \neq \omega_0 = 4$ *and because the differential operator on the left only has even derivatives, we may use the undetermined coefficients guess*

$$x_P(t) = A\cos(4.5\,t).$$

(ii) Because $\omega \approx \omega_0$ *the general solutions* $x(t) = x_P(t) + x_H(t)$ *will exhibit* beating.

(3 points)