Name________________________
Student I.D.___________________

Math 2250-010
Quiz 6 Take-home
February 28, 2014
Due at the start of class on Monday

1a) Consider the differential equation for $y(x)$

$$y''(x) + 6\,y'(x) + 8\,y(x) = 0\,.$$

Find the general solution to this homogeneous differential equation. Hint: the solution space has a basis consisting of exponential functions.

(4 points)

the characteristic polynomial (obtained by seeing which functions $y = e^{rx}$ can solve the DE) is

$$p(r) = r^2 + 6\,r + 8 = (r+2)(r+4)$$

which has roots $r = -2, -4$. Thus a basis for the solution space consists of the two functions $y_1(x) = e^{-2x}, y_2(x) = e^{-4x}$, and the general homogeneous solution is the collection of all linear combinations of the basis functions,

$$y(x) = c_1 e^{-2x} + c_2 e^{-4x}$$

(with $c_1, c_2 \in \mathbb{R}$).

1b) Verify that $y(x) = 4\,x - 3$ is a solution to the inhomogeneous differential equation

$$y''(x) + 6\,y'(x) + 8\,y(x) = 32\,x.$$

(1 points)

We verify that this function $y(x)$ makes the differential equation true: For $y(x) = 4\,x - 3, y'(x) = 4, y''(x) = 0$ so

$$y'' + 6\,y' + 8\,y = 0 + 6\cdot 4 + 8(4\,x - 3) = 24 + 32\,x - 24 = 32\,x$$

which is the correct right hand side, so the DE is true.

1c) Combine your work from a, b to deduce the general solution to the inhomogeneous DE in b. Then use this general solution to solve the initial value problem

$$y''(x) + 6\,y'(x) + 8\,y(x) = 32\,x$$
$$y(0) = -2$$
$$y'(0) = 0.$$

(5 points)

We know that the general inhomogeneous solution is of the form $y = y_P + y_H$, i.e.

$$y(x) = 4\,x - 3 + c_1 e^{-2x} + c_2 e^{-4x}$$
$$\Rightarrow y'(x) = 4 - 2\,c_1 e^{-2x} - 4\,c_2 e^{-4x}$$

We wish to solve

$$y(0) = -2 = -3 + c_1 + c_2$$
$$y'(0) = 0 = 4 - 2\,c_1 - 4\,c_2$$

i.e.

$$c_1 + c_2 = 1$$

$$c_1 + 2\,c_2 = 2$$

Subtracting equations implies $c_2 = 1$, *so* $c_1 = 0$.

$$y(x) = 4\,x - 3 + e^{-4\,x}$$

```
> with(DEtools):
  dsolve({y''(x) + 6·y'(x) + 8·y(x) = 32·x, y(0) = -2, y'(0) = 0});
      # I checked all my work by hand, e.g. the initial values, but this is a tech check
```

$$y(x) = -3 + 4\,x + \mathrm{e}^{-4\,x} \qquad (1)$$

```
>
```