

Name _____
Student I.D. _____

Math 2250-010
Quiz 5
February 21, 2014

1a) Define what a linear combination of vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ is.

(2 points)

any sum of scalar multiples of those vectors, i.e. any vector \mathbf{v} that can be written as

$$\mathbf{v} = c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \dots + c_n \mathbf{v}_n$$

1b) Define what the span of a collection of vectors $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$ is.

(2 points)

the collection of all linear combinations, i.e.

$$\{c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \dots + c_n \mathbf{v}_n \text{ such that } c_1, c_2, \dots, c_n \in \mathbb{R}\}$$

2) The span of the vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

is a plane in \mathbb{R}^3 . Find for which $[x, y, z]^T$ the system

$$c_1 \mathbf{u} + c_2 \mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

can be solved for c_1, c_2 , in order to find the implicit equation $ax + by + cz = d$ of this plane. Hint: write down the augmented matrix for this system with unknowns c_1, c_2 , reduce it, and interpret your results.

(6 points)

The linear system above has augmented matrix

$$\left[\begin{array}{cc|c} 1 & 2 & x \\ 2 & 3 & y \\ 3 & 4 & z \end{array} \right]$$

which we reduce:

$$-2 R_1 + R_2 \rightarrow R_2; -3 R_1 + R_3 \rightarrow R_3:$$

$$\left[\begin{array}{cc|c} 1 & 2 & x \\ 0 & -1 & -2x + y \\ 0 & -2 & -3x + z \end{array} \right]$$

$$-2 R_2 + R_3 \rightarrow R_3:$$

$$\left[\begin{array}{cc|c} 1 & 2 & x \\ 0 & -1 & -2x + y \\ 0 & 0 & (4x - 2y) - 3x + z \end{array} \right]$$

Thus the system will be consistent if and only if the lower right entry is zero, i.e. (after simplifying)

$$x - 2y + z = 0$$

which is therefore the implicit equation of the plane spanned by the two vectors $[1, 2, 3]^T$, $[2, 3, 4]^T$.