Quiz 4 SOLUTIONS
February 7, 2014

1a) Consider the following system of equations

$$\begin{aligned} 2x + y \quad &= 3 \\ 3x + \quad 3z &= 6 \\ -x - 2y + 3z &= 0 \end{aligned}$$

Exhibit the augmented matrix corresponding to this system, compute its reduced row echelon form, and find the solution set to the system.

(6 points)

The augmented matrix is

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 3 & 0 & 3 & 6 \\ -1 & -2 & 3 & 0 \end{array}\right]$$

which we reduce: $-R_3 \to R_1, R_1 \to R_3, \frac{R_2}{3} \to R_2$:

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 3 \end{array}\right]$$

$-R_1 + R_2 \to R_2; -2\,R_1 + R_3 \to R_3$:

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & -2 & 4 & 2 \\ 0 & -3 & 6 & 3 \end{array}\right]$$

$\frac{R_2}{-2} \to R_2, \frac{R_3}{-3} \to R_3$:

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & -2 & -1 \end{array}\right]$$

$-R_2 + R_3 \to R_3$:

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$-2\,R_2 + R_1 \to R_1$:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Thus, backsolving: $z = t \in \mathbb{R}$, $y = -1 + 2\,t$, $x = 2 - t$, or in vector form

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 - t \\ -1 + 2t \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}.$$

(This is a parametric description of a line passing through $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ with tangent vector $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$.)

1b) Interpreting the solution set of each single equation above as a plane in $\mathbb{R}^3$, what geometric configuration corresponds to the solution set of the system of 3 equations above?

Three planes intersecting in a line

(2 points)

2) Consider the matrix equation

$$\begin{bmatrix} 1 & -2 & 3 & 4 \\ 5 & -2 & 7 & 3 \\ 4 & 0 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

with $\underline{b} \neq \underline{\mathbf{0}}$. Without trying to find the solution set explicitly, explain which of the following three outcomes are possible for the solution set, just based on the number of equations, the number of unknowns, and the right hand side: (a) no solutions; (b) exactly one solution; (c) infinitely many solutions.

(2 points)

(a) no solutions is possible - if the coefficient matrix reduces to have a row of zeros on the bottom then inconsistent systems are possible.
(b) exactly one solution is not possible: if solutions exist there will be at least one free parameter, since at least one column will not have a leading one.
(c) infinitely many solutions is possible - in fact as long as the system is consistent it will automatically have infinitely many solutions, since there will be at least one free parameter.