Math 2250-10
Quiz 2 SOLUTIONS
January 17, 2014

1a) Find the general solution to the differential equation for $y(x)$

$$y'(x) = -2y + 8$$

using the method for separable differential equations.

(5 points)

$$\frac{dy}{dx} = -2y + 8 = -2(y - 4)$$

$$\frac{dy}{y - 4} = -2\,dx$$

(unless $y_0 = 4$, in which case the solution is $y(t) \equiv 4$)

$$\int \frac{dy}{y - 4} = \int -2\,dx$$

$$\ln|y - 4| = -2x + C_1$$

exponentiate:

$$|y - 4| = e^{-2x} e^{C_1}$$

Since $y(t)$ is differentiable it's also continuous, so $|y(t) - 4|$ also is - and is never zero. Thus

$$y - 4 = Ce^{-2x}$$

where $C = e^{C_1}$ or $C = -e^{C_1}$. Thus

$$y(x) = 4 + Ce^{-2x} \ .$$

(By letting $C = 0$ we also pick up the singular solution $y = 4$.)

1b) Solve the same differential equation

$$y'(x) = -2y + 8$$

using the method for linear differential equations.

(5 points)

$$y'(x) + 2y(x) = 8$$

$$e^{2x}(y' + 2y) = 8e^{2x}$$

$$\frac{d}{dx} e^{2x} y(x) = 8e^{2x}$$

$$e^{2x} y = \int 8e^{2x} dx = 4e^{2x} + C.$$

Divide both sides by the integrating factor to solve for $y(x)$:

$$y = 4 + Ce^{-2x}$$