Math 2250-10
Quiz 1 SOLUTIONS
January 10, 2014

1a) Consider the differential equation for $y = y(x)$:

$$y' = 2y + 6$$

Show that the functions $y(x) = C e^{2x} - 3$ *solve this differential equation.*

(2 points)

We show that the given functions $y(x)$ make the differential equation a true equation. For $y(x) = C e^{2x} - 3$ the left side of the DE is

$$y'(x) = 2 C e^{2x}.$$

The right side of the DE is

$$2y + 6 = 2(C e^{2x} - 3) + 6 = 2 Ce^{2x} - 6 + 6 = 2 Ce^{2x}.$$

Since the left side and right side of the DE evaluate to the same function, it is a true equation and the given $y(x)$ are solutions.

1b) Find a solution to the initial value problem

$$y' = 2y + 6$$
$$y(0) = 5$$

For a trial solution $y(x) = C e^{2x} - 3$ we set $y(0) = 5$ and solve for C:

$$y(0) = C - 3 = 5 \Rightarrow C = 8.$$

So a (the) solution to the IVP is

$$y(x) = 8e^{2x} - 3\ .$$

(2 points)

2) The brakes of a car are applied when it is moving 40 $\frac{m}{s}$*, and they provide a constant deceleration of 8* $\frac{m}{s^2}$*. How far does the car travel before coming to a stop? Hint: find formulas for the velocity and position functions, and use those to answer the problem.*

(6 points)

We learned a formula in class that the distance traveled is

$$x_{max} = \frac{v_0^2}{2a} = \frac{40^2}{16} = \frac{1600}{16} = 100\ m$$

Alternately (and what we expected most students to do) you can integrate the acceleration to get velocity and position functions, and then work out the answer:

$$v'(t) = -8$$
$$\Rightarrow v(t) = \int -8\ dt = -8\,t + C = -8\,t + v_0 = -8\,t + 40\ .$$
$$\Rightarrow x(t) = \int -8\,t + 40\ dt = -4\,t^2 + 40\,t + x_0.$$

We may set $x_0 = 0$, so

$$x(t) = -4t^2 + 40t.$$

The maximum x will occur when $v(t) = 0 = -8t + 40$, i.e. at $t = 5$ sec. Then the distance traveled is

$$x(5) - x(0) = -4 \cdot 5^2 + 40 \cdot 5 = -100 + 200 = 100\ m.$$