**Part 1:** This is part of the homework that is due on Friday at the start of class.

1. (This is homework problem w5.1) Find the equation of the parabolic graph that passes through the points $(1, 0), (2, 3)$ and $(4, 3)$, as follows: Exhibit and solve the system of three equations in the three unknowns a, b, c for the unknown parabola $y = ax^2 + bx + c$. Use Gaussian elimination to compute the reduced row echelon form of the augmented matrix, in order to find a, b, c. Sketch by hand the resulting parabola and its vertex, together with the three points it passes through.

Math 2250 Lab 5   Name/Unid: _____

1. The balancing of chemical reactions always leads to systems of linear equations. The equations are often "sparse" enough that the method of substitution will work well, but Gaussian elimination is guaranteed to work.

   (a) The complete combustion of propane C_3H_8 with oxygen O_2 results in carbon dioxide CO_2 and water H_2O, in such a way that the total numbers of each element are the same before and after the reaction:

   $$a(C_3H_8) + b(O_2) \rightarrow c(CO_2) + d(H_2O).$$
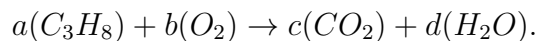
   The coefficients a, b, c, d for the numbers of each molecule need to be found from the three equations equating the number of C, H, O atoms on each side. For example, equating the number of oxygen atoms yields the equation

   $$2b = 2c + d \Rightarrow 2b - 2c - d = 0.$$

   Find the other two equations, and then use Gaussian elimination to find the solutions to this homogeneous system of three equations in four unknowns. Pick the free parameter in the solution so that a, b, c, d are each positive integers having no common factors. This will give you the usual balancing equation for complete combustion of propane.

(b) If there is not enough oxygen for the amount of propane, combustion will be incomplete and the intermediate product carbon monoxide CO will be created but not completely converted into carbon dioxide CO_2. (This is why cooking with propane-fueled grills is usually done outdoors, because CO is potentially toxic.) The more complicated partial combustion equation is

$$a(C_3H_8) + b(O_2) \rightarrow c(CO_2) + d(H_2O) + e(CO).$$

If we set $a = 1$ this equation becomes

$$(C_3H_8) + b(O_2) \rightarrow c(CO_2) + d(H_2O) + e(CO).$$

Solve for c, d, e in terms of the parameter b. Note that all four unknowns must be non-negative, so that b can only be in a certain interval. Discuss the range of outcomes as b varies in this interval.

2. A commercial customer has ordered 104 gallons of paint that contains an equal amount of red paint, yellow paint, green paint, and blue paint, and hence could be prepared by mixing 26 gallons of each color. However, the store wishes to prepare this order by mixing four types of paint that are already available in large quantity:

Paint A is a mixture of 50.5% red, 9.5% yellow, 20% green, and 20% blue paint;
Paint B is a mixture of 10% red, 2.5% yellow, 70% green, and 17.5% blue paint;
Paint C is a mixture of 5% red, 20% yellow, 15% green, and 60% blue; and
Paint D is a mixture of 15.5% red, 65.5% yellow, 4% green, and 15% blue.

How many gallons of paints A,B,C,D are needed to prepare the customer's order? Write out a system of equations that represents the information given, then solve that system using Maple, Matlab, Wolfram alpha, or any other favorite mathematical software.

3. Consider the cascade of four tanks in the following figure with the volumes (in L) as $V_1 = 20$, $V_2 = 30$, $V_3 = 10$ and $V_4 = 40$. The flow rates (in L/m) are indicated in the figure (e.g. $r_{2,1}$ is the flow rate from tank 1 to tank 2). The functions $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$ (in g) represent the amount of salt in the tanks 1, 2, 3, and 4 at time t respectively. Suppose the concentration of the solute flowing into tank 1 is 1 g/L.
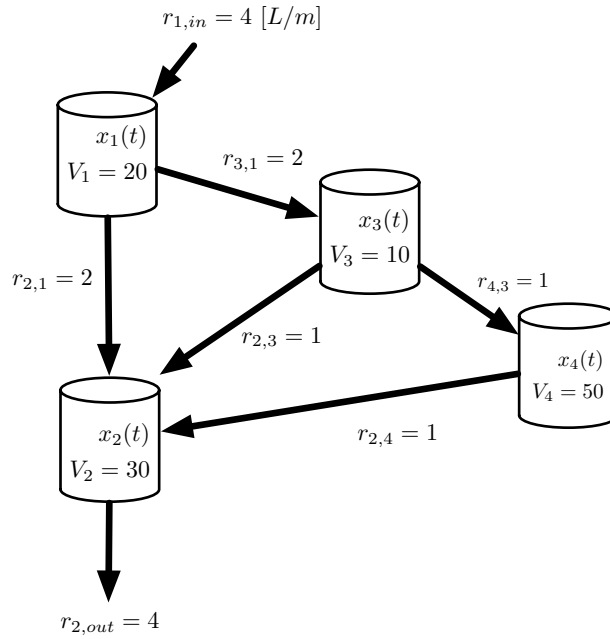
(a) Use input-output modeling to express $x_1'(t)$, $x_2'(t)$, $x_3'(t)$ and $x_4'(t)$ in terms of $x_1(t)$, $x_2(t)$, $x_3(t)$ and $x_4(t)$.

(b) Rewrite the differential equations in part (a) in matrix-vector form, i.e. find a 4×4 matrix $\mathbf{A}$ and a constant vector $\mathbf{v}$ so that

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix} = \mathbf{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \mathbf{v}.$$

(c) An "equilibrium solution" for this system of differential equations would be one in which all of the solute amounts $x_1(t), x_2(t), x_3(t), x_4(t)$ are constant functions of time, i.e. the time derivative of the vector of solute amounts is zero. Use technology or work by hand to solve the matrix equation

$$\mathbf{0} = \mathbf{A}\mathbf{x} + \mathbf{v}$$

for the vector $\mathbf{x}$ of solute amounts, in order to find this equilibrium solution.

(d) Given that you know the input solute concentration coming into the cascade is 1 g/L, how could you have deduced the (constant) equilibrium solution directly, just based on the fact that liquid eventually gets flushed through the system?

4. (a) Use Matlab or Maple to compute both the determinant and the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 2 & 1 & 4 \\ 4 & 2 & 3 & 1 \end{pmatrix}$$

(b) Augment your matrix with the identity matrix. Do this by simply creating a 4x8 matrix with the first 4 columns making the original matrix and the last 4 the identity matrix. Using Maple or Matlab put this matrix into *reduced row echelon form* and compare the result with your answer above.

(c) To triple check that the inverse matrix you found is in fact the inverse of the original matrix, multiply it with the original matrix. What do you get?

Some Maple and Matlab Examples

Here the matrix $A = \begin{pmatrix} 2 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ will be used as an example.

When working with matrices in Maple you should first call the *LinearAlgebra* package. This is done by running the command '*with(LinearAlgebra):*'. This will only need to be done once in your Maple document and run it before your commands involving matrices are run.

Creating a matrix

In Maple you would type '*A:=Matrix([[2,2,3],[4,5,6],[7,8,9]])*'.

The *Matrix* command on the right hand side is what tells Maple to turn the data afterwards into a matrix. The *:=* stores that matrix into the variable A. Using variables like this is very useful when you plan on using the matrix you have entered more than once since after doing this you can refer to that matrix by simply typing A.

To do the same thing in Matlab you would type '*A=[2 2 3; 4 5 6; 7 8 9]*'.

The rest of these examples will be done with the assumption that the matrix is stored into the variable A.

Reduced row echelon form	Augment the two matrices A and B
Maple: *ReducedRowEchelonForm(A)*	Maple: $\langle A|B \rangle$
Matlab: *rref(A)*	Matlab: $[A\ B]$

Computing the inverse	Creating an nxn identity matrix
Maple: A^{-1}	Maple: *IdentityMatrix(n)*
Matlab: *A^(-1)* or *inv(A)*	Matlab: *eye(n)*

Computing the determinant
Maple: *Determinant(A)*
Matlab: *det(A)*

Multiplying two matrices A and B
Maple: *A.B*
Matlab: *A*B*