

Part 1: This is part of the homework that is due on Friday at the start of class.

1. (This is 2.3.10, as modified in our homework.) A woman bails out of an airplane at an altitude of 10,000 feet, falls freely for 25sec, then opens her parachute. How long will it take her to reach the ground? Assume linear air resistance $\rho v \frac{ft}{sec}$, taking $\rho = 0.15$ without the parachute and $\rho = 1.5$ with the parachute. These steps might help organize your work:
 - (a) Write down the differential equation and initial condition that describes the velocity v before she opens the parachute, and solve the IVP.
 - (b) Find the velocity at time 1 s (see subsequent lab problem), the velocity when she just opens the parachute, and the terminal velocity.
 - (c) Find a formula for her height $y(t)$ above the ground, and use this to compute her height when the parachute opens.

- (d) Solve a second initial value problem to find a formula for the new velocity function, and the new height function, after the parachute opens.
- (e) Use a numerical "solve" command to find out when her height is zero (because the time cannot be solved for algebraically). Then use all of your work to answer the original question about how long she is in the air.

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Part 2: These lab problems are due Wednesday February 5 at the start of class.

1. (15 points) Consider the same IVP as in Part I, parts 1ab. Apply the following methods to approximate the solution function $v(t)$ on the time interval $0 \leq t \leq 1$. Do these computations by hand (using a calculator or other technology to do each step, but not using programmed code). You may wish to use programmed code (e.g. modifications of the Matlab and Maple files posted on our homework page) to check your work before you hand it in.
 - (a) Euler Method: use step size $h = 0.5$, i.e. 2 time steps.
 - (b) Improved Euler Method: use step size $h = 0.5$ again.

- (c) Runge Kutta Method: use step size $h = 1.0$, i.e. just 1 time step.
- (d) Compare these approximations for $v(1)$ with the exact solution at $t = 1$ and comment about the relative accuracy of the three techniques, by computing the relative errors

$$\left| \frac{v_{approx} - v_{exact}}{v_{exact}} \right|$$

in each case. (The step sizes are so large that you don't expect any of the techniques to be extremely accurate.)

2. (20 points) If we drop a package from a helicopter with a parachute attached, the wind resistance provided by the parachute is not really a linear function of the velocity. Assume for a certain type of parachute the acceleration equation is modeled by

$$v'(t) = 32 - 0.18v - 0.16v^{1.6}$$
$$v(0) = 0$$

We are assuming velocity is in ft/s, and since the acceleration of gravity is displayed as $+32\frac{ft}{s^2}$ we have chosen the positive direction to be pointing down.

- (a) What is the terminal velocity in this model? Notice that the "slope function" $32 - 0.18v - 0.16v^{1.6}$ for the graph of $v(t)$ is continuous and decreasing function of v for $v > 0$ that its value is 32 when $v=0$, and that its limit as $v \rightarrow \infty$ is $-\infty$. Thus the slope function has exactly one root and you can find it with the Matlab (or other software)"solve" command.
- (b) This is a differential equation which does NOT have an elementary solution. Use Matlab, Maple, or other software to estimate the solution values at $t = 3$ and $t = 6$ seconds with the methods Euler, Improved Euler and Runge Kutta with step sizes $h = 0.3$ and $h = 0.03$. There is sample Matlab and Maple code posted on our homework page, which you may modify as needed. If you prefer to use an alternate software package such as Python or Mathematica, this is also O.K.
- Write the approximate solution values for each method with each h and each time $t=3$ and $t=6$.
 - Plot your results and comment about apparent accuracy of the three methods with these different choices of time step size.

- (c) Since we don't know the exact solution function in this problem, there is not a quick direct way to see how accurate our numerical approximate solutions are. There are error estimates that some of you will learn in later numerical analysis classes (analogous to error estimates you may have learned for Taylor series), but for the purposes of this lab undertake the following crude test: Compare the numerical approximations for velocity at $t = 3$ and $t = 6$, for Runge-Kutta with $h = 0.3$ and $h = 0.03$. If these approximations are close (and the approximate solution graphs appear to line up exactly), then it's likely that they are also close to the exact solution.
- (d) Based on your work above, approximately what fraction of the terminal velocity is attained at $t = 3$ and $t = 6$ seconds?
- (e) What does the apparent horizontal asymptote on the plots mean? What happens as $t \rightarrow \infty$ if you change the initial condition to $v(0) = 5$ and $v(0) = 40$? Explain with a phase diagram.