

Part 1: These problems are part of the homework that is due on Friday at the start of class.

1. (this is 2.1.3) Use separation of variables and partial fractions to solve following initial value problem for $x(t)$:

$$\frac{dx}{dt} = 1 - x^2$$

$$x(0) = 3$$

.

2. (this is w3.2) Consider the differential equation

$$\frac{dx}{dt} = 4x^4 - x^2$$

- (a) Find the equilibria and draw the phase portrait.
- (b) Classify the equilibria as stable, asymptotically stable, or unstable (possibly one-sided stable).
- (c) Use dfield to sketch the slope field and representative solution graphs, including graphs of the equilibrium solutions. Include this plot in your homework.

Part 2: These lab problems are due Wednesday January 29 at the start of class

1. (15 points) Consider a bioreactor used by a yogurt factory to grow the bacteria needed to make yogurt. The growth of the bacteria is governed by the logistic equation

$$\frac{dP}{dt} = k \cdot P(M - P)$$

where P is the population in millions and t is the time in days. Recall that M is the carrying capacity of the reactor and k is a constant that depends on the growth rate.

- (a) Through observation it is found that after a long time the population in the reactor stabilizes at 100 million bacteria, and that when the population of the reactor is 20 million the population increases at a rate of 15 million bacteria per day. From this, find k and M in the governing equation.
- (b) If the colony starts with a population of 40 million bacteria, how long will it take for the population to reach 65% of the carrying capacity?
- (c) Suppose the factory harvests bacteria from the reactor once a week. The harvesting process takes a day, during which the reactor is not operational, leaving 6 days per week for the bacteria to grow in the reactor. The factory wants to maximize the amount of bacteria grown during these 6 days. To achieve this, $\frac{dP}{dt}$ should be at its maximum 3 days after harvesting. What initial population (after harvesting) gives the most growth over the 6-day period? What is the population change during that time?
- (d) Suppose the reactor is modified to allow for continual harvesting without shutting down the reactor. Let h be the rate at which the bacteria are harvested in millions per day. Write down the new differential equation governing the bacteria population. What is the maximum rate of harvesting h that will not cause the population of bacteria to go extinct? (Below this rate there will always be a stable equilibrium point where P is positive).

2. (10 points) Suppose you wish to dispose nuclear waste of by placing it in sealed drums and dropping the drums into the ocean. Each filled drum weighs 640 lb on land and has a volume of 8 ft³. The force equation for a drum falling though water is:

$$m \frac{dv}{dt} = -W + B + F_R$$

where W is the weight of the drum, B is the buoyant force equal to the weight of the water displaced by the drum (the density of water is 62.5 lb/ft³), and F_R is the force of water resistance, known to be 1 lb for every ft/s that the drum is moving. The drums may burst if they hit the ocean floor at a speed higher than 120 ft/s. If no initial velocity is given to the drums when they are dropped, what is the deepest water that the drums can safely be dropped into? (Hint: recall in the English system of units, weight in pounds is a force. Thus, since $w = mg$ and since the numerical value of $g = 32 \frac{ft}{sec^2}$, the mass in slugs will be $\frac{1}{32}w$.)