

Part 1: These problems are part of the homework that is due on Friday at the start of class.

1. This is w2.2 from this week's homework set.

In homework problem 1.1.29 last week you showed that if a function  $y = y(x)$  has the property that every straight line normal to the graph  $y = y(x)$  passes through the point  $(0, 1)$ , then  $y(x)$  satisfies the differential equation

$$\frac{dy}{dx} = -\frac{x}{y-1}$$

- (a) Use the existence-uniqueness theorem to verify that the initial value problem for this differential equation, with  $y(3) = -3$ , has a unique solution.
- (b) Use dfield to create a slope field for this differential equation inside the rectangle  $-6 < x < 6, -5 < y < 7$ . What do you notice about the apparent shapes of the solution graphs? Does this make sense based on the original geometric description that led to this differential equation? Explain.
- (c) Solve the initial value problem in part a, to explicitly find  $y(x)$ . What is the largest  $x$ -interval on which  $y(x)$  can be defined as a differentiable function? Explain.

2. This is problem 1.4.42 from the textbook.

A certain moon rock was found to contain equal numbers of potassium and argon atoms. Assume that all the argon is a result of radioactive decay of potassium (its half-life is about  $1.28 \times 10^9$  years) and that one of every nine potassium atom disintegrations yields an argon atom. What is the age of the rock, measured from the time it contained only potassium?

Part 2: These three lab problems are to be handed in before the start of class on Wednesday January 22. Explain all work carefully and neatly.

1. (20 points) A 22-year-old woman accepts an engineering position with a starting salary of \$70,000 per year. Her salary  $S(t)$  increases exponentially at a continuous rate of 4% a year, so that  $S(t) = 70e^{t/25}$  thousand dollars per year, after  $t$  years. To save for retirement, she deposits 10% of her salary continuously into a retirement account, which accumulates interest continuously at an annual rate of 3%. Let  $A(t)$  be the amount in the retirement account after  $t$  years, with  $A(0) = 0$  thousands of dollars at the time she begins her new job.
  - (a) Estimate  $\Delta A$  in terms of  $\Delta t$ , to derive the differential equation satisfied by  $A(t)$ .
  - (b) Compute the amount of money that she will have in her retirement account if she retires at age 70 (recall that she is 22 years old at year  $t = 0$ ).

2. (20 points) Consider the set up consisting of two brine tanks where water cascades from tank 1 down into tank 2, and then out of tank 2. (see figure 1.5.5 in your text book). Suppose that when the system is set into motion, the first tank contains 100 gallons of brine, while the tank below it contains 120 gallons of pure water. When the system starts, fresh water is pumped into tank 1 at a constant rate of 2 gal/min. At the same time the brine solution in tank 1 drains into tank 2 at a rate of 2 gal/min, and tank 2 drains at this same rate. Hence the volumes in both tanks stay constant. Assume that the solutions in each tank remain well-mixed, so that although the salt concentrations are changing in time, the concentration of salt leaving each tank equals the average concentration in that tank.
- (a) Suppose that  $x(t)$  is the amount of salt in tank 1 at time  $t$ . If the first tank originally contains 80 lbs of salt, derive and solve an initial value problem to find a formula for  $x(t)$
  - (b) Suppose that  $y(t)$  is the amount of salt in tank 2 at time  $t$ . From conservation laws, derive the differential equation that  $y(t)$  must satisfy (this should involve  $x(t)$ , the amount of salt in the first tank).
  - (c) Suppose that  $y(t)$  satisfies  $\frac{dy}{dt} = \frac{x}{50} - \frac{y}{60}$ . Solve for  $y(t)$ .
  - (d) Using Matlab, Maple, or other technology, create a graph that shows the amount of salt in each tank for the first two hours after the system is set in motion.

3. (20 points) Use the *dsolve* command in Maple to solve the differential equation

$$(x + e^y)y' = e^{-y} - 1$$

This is done by first loading the differential equations package, and then using the *dsolve* command. For details on using Maple, read the introductory notes and commands posted at the URL

<http://www.math.utah.edu/~korevaar/2250spring14/2250hw.html>

or consult the Maple help files, or use google. You should get two concise solution formulas (if the formula you get is multiple lines long it is likely incorrect). Pick one of these and use Maple to verify that it indeed solves the differential equation. You should print off the Maple code you used and attach it to this worksheet (if that is not possible you can copy it by hand onto the worksheet).