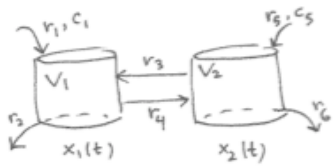
Math 2250-010
Week 13 Lab

This week's lab is a chance to work together on the homework problem w13.4, which is due as part of the regular homework on Monday April 14, by 5:00 p.m. This problem ties together key ideas related to first order systems of linear differential equations.

<u>**w13.4**</u> Consider a general input-output model with two compartments as indicated below. The compartments contain volumes V_1, V_2 and solute amounts $x_1(t), x_2(t)$ respectively. The flow rates (volume per time) are indicated by r_i , $i = 1..6$. The two input concentrations (solute amount per volume) are c_1, c_5.



<u>**a)**</u> What equalities between the flow rates guarantee that the volumes V_1, V_2 remain constant?

<u>**b)**</u> Assuming the equalities in <u>**a**</u> hold, what first order system of differential equations governs the rates of change for $x_1(t), x_2(t)$?

c) Suppose $r_2 = r_4 = r_6 = 100$, $r_3 = r_5 = 200$, $r_1 = 0 \ \dfrac{gal}{hour}$; $c_1 = 0$, $c_5 = 0.3 \ \dfrac{lb}{gal}$; $V_1 = V_2 = 100\ gal$.
Verify that the constant volumes are consistent with the rate balancing required in **a**. Then show that the general system in **b** reduces to the following system of DEs for the given parameter values:

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 60 \end{bmatrix}.$$

d) Solve the initial value problem for **c**, assuming there is initially no solute in either tank. Hint: Find the homogeneous solution; then find a particular solution which is a constant vector; and then use $\underline{\boldsymbol{x}} = \underline{\boldsymbol{x}}_P + \underline{\boldsymbol{x}}_H$ to solve the IVP.

e) Re-solve the initial value problem in **c** using Laplace transforms.
f) Check your answer to **d** (and **e**) with technology, and hand in a copy of this verification. For example, in Maple, the "dsolve" command can solve systems of differential equations as well as single differential equations. (See the list of Maple commands on our homework page.)