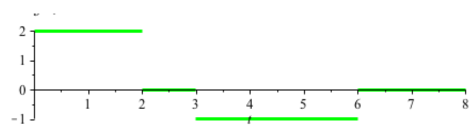
Math 2250 Lab 12 Name/Unid: ______________________________

Part 1: This is part of the homework that is due on Monday by 5:00 p.m.

1. (This is homework problem w12.4.) In a week 11 homework problem you solved the following Laplace transform problem using the definition of Laplace transform. This week, write $f(t)$ as a linear combination of unit step functions in order to find (the same) Laplace transform. Here's the graph of f, which is zero for $2 \le t < 3$ and $t \ge 6$:



2. (This is problem w12.5 in this week's homework set.)
   (a) Find the convolution $f * g(t)$ for $f(t) = t$ and $g(t) = t^2$ and use the Laplace transform table to verify that $\mathcal{L}\{f*g(t)\}(s) = F(s)G(s)$ in this case. Be clever about which order you write the f, g terms in the convolution integral, as convolution is commutative and one way makes for an easier integral.
   (b) Repeat part (a) for $f(t) = t, g(t) = sin(kt)$.

1. (10 points) The following problem illustrates how, by studying how a mechanical or electrical system responds to various different input forcing functions, one can deduce information about the components of the system.

   Consider a forced oscillator differential equation for $x(t)$:

$$ax''(t) + bx'(t) + cx(t) = f(t)$$

   Assume the parameters a, b, c are initially unknown. Assume that you do know from observational measurements and curve fitting, that when the initial conditions are $x(0) = 0, x'(0) = 0$, then the outputs $x(t)$ from this system (approximately) satisfy

$$x(t) = \int_0^t e^{-2\tau} \sin(\tau) f(t-\tau) d\tau$$

   no matter what the forcing function $f(t)$ is. Notice that this is a Laplace transform convolution integral. The value of $x(t)$ depends on the values of the forcing function $f(r)$ for $0 \leq r \leq t$, $(r = t - \tau)$, and on the value of so-called "weight function" $w(\tau) = e^{-3\tau} \sin(\tau)$, also for the previous times $0 \leq \tau \leq t$, in some precise but convoluted way.

   (a) The Laplace transform $W(s)$ of weight function $w(t) = e^{-2t} \sin(t)$ is called the "transfer function." What is the formula for $W(s)$?

   (b) Since $x(t)$ is given by the convolution integral of w and f, its Laplace transform $X(s)$ must be the product $W(s)F(s)$. On the other hand, since $x(t)$ solves the forced oscillation initial value problem, you know how to find its Laplace transform $X(s)$ by transforming the differential equation. Equate these two formulas for $X(s)$ to deduce the parameter values a, b, c in the inhomogeneous ODE describing your mechanical/electrical system. If this was a mass,spring,damper system, what would be the values of m, c, k? If it was an electrical system, what would the values of L, R, C be?

2. (10 points) In practice, convolution integrals are often used to obtain solutions of initial value problems (IVPs), particularly those with complicated forcing functions $f(t)$. As seen from methods in problem (1), we can use the convolution integral

$$x(t) = \int_0^t w(\tau) f(t-\tau) d\tau.$$

To see how this works in practice, consider what happens to an IVP with weight function

$$w(t) = e^{-\frac{t}{6}} \sin(3\pi t)$$

when it is forced with a logistic function

$$f(t) = \frac{1}{1 + e^{-k(t-b_1)}}$$

(a) Set $k = 6$ and b_1=1. Using the provided code *convolvetosolve2250.m*, plot the solution $x(t)$ along with the logistic forcing $f(t)$ and the DE kernel $w(t)$.

(b) Solve again using $k = 12$ and plot your results. What effect does the change in k have on your forcing function $f(t)$? How does this change the solution of your DE?

(c) Instead of forcing with the logistic function, change $f(t)$ to be a step function (provided in code). Compare the solution of your DE to what you saw in (a) and (b).

(d) Imagine $x(t)$ is modeling the flow of current in a power grid at time t. If, for example, too many air conditioning units turn on at once, there may be a current surge that would blow the transformer. Is (a), (b), or (c) most likely to model this situation and why?

3. (10 points) Consider an RLC Circuit with $R = 210\ \Omega$, $L = 1$ H, $C = 0.002$ F, and a battery supplying $e_0 = 190$. Additionally, suppose I(0)=0, I'(0)=190. At time t=0 the switch is closed and at time t=1 it is opened and left open. This circuit can be modeled as:

$$LI'' + RI' + \frac{1}{C}I = e'(t)$$

where $e'(t) = -190\delta(t-1)$.

(a) The delta function $\delta(t-a)$ satisfies

$$\int_0^{\infty} f(t)\delta(t-a)dt = f(a)$$

Using the definition of Laplace Transform, find $\mathcal{L}\{\delta(t-a)\}$.

(b) Find the resulting current I(t) in the circuit using Laplace Transform methods.