Math 2250 Lab 10   Name/Unid: _____

These problems are w10.2, w10.3, and w10.4 respectively in the homework that was originally due on Friday March 21, but that is now due on Monday March 24. There are no separate lab problems this week (no "part 2").

Consider a mass-spring-dashpot system with an additional external force $F(t)$ being applied to the mass. In particular we consider a periodic external force $F(t) = F_0\cos(\omega t)$. As we know, this system is governed by the following differential equation for the displacement $x(t)$ from equilibrium:

$$m \cdot x''(t) + c \cdot x'(t) + k \cdot x(t) = F_0\cos(\omega t)$$

Take $m = 1$ kg, $k = 0.64$ N/m, and $F_0 = 2$ N. Also the system will always start at rest, i.e. $x(0) = 0$, $x'(0) = 0$ The damping coefficient c will be modified in different problems, as will the angular frequency ω of the driving force.

1. Consider the configuration above, in the undamped case $c = 0$. In particular consider the initial value problem

$$x''(t) + 0.64 \cdot x(t) = 2\cos(\omega t)$$
$$x(0) = 0$$
$$x'(0) = 0$$

(a) What is the natural angular frequency ω_0 (for the unforced problem) in this differential equation? **Hint:** the natural frequency is defined to be the angular frequency for the solutions to the unforced and undamped differential equation, which in this case is the DE

$$x''(t) + 0.64 \cdot x(t) = 0.$$

(b) Assume that $\omega \neq \omega_0$. Use the method of <u>undetermined coefficients</u> to solve for the particular solution $x_P(t)$. Then use $x(t) = x_P(t) + x_H(t)$ to solve the initial value problem for $x(t)$, where $x_H(t)$ is a solution to the corresponding homogeneous (unforced) equation. Use a technology to check that this solution is correct.

(c) Write down the specific case of the solution $x(t)$ (found in part b) for $\omega = 0.7$. Compute the period of this solution, which is a superposition of two cosine functions. Use technology to graph one period of the solution. What phenomenon is somewhat exhibited by this solution?

(d) Solve the IVP when $\omega = \omega_0$. Use the method of <u>undetermined coefficients</u> to solve for a new particular solution, then use $x(t) = x_P(t) + x_H(t)$ to solve the initial value problem for $x(t)$. Graph this solution on the interval $0 \leq t \leq 60$ seconds. What phenomenon is exhibited by this solution?

2. Consider the same mass-spring-dashpot system as above, except with $c = 2$ kg/s, and $\omega = 0.8$. This gives us the differential equation

$$x''(t) + 2x'(t) + 0.64x(t) = 2\cos(0.8 \cdot t)$$

(a) Use the method of undetermined coefficients to find a particular solution $x_P(t)$ to this differential equation.

(b) Use the particular solution $x_P(t)$ found in part (a) and the solution $x_H(t)$ to the corresponding homogeneous equation to write down the general solution to this differential equation. Identify the "steady periodic" and "transient" parts of this general solution.

(c) Solve the initial value problem for this differential equation above, with $x(0) = 0$, $x'(0) = 0$. (You may use technology to do this, which will also check your work in parts a,b.)

(d) Graph, on a single plot, the steady periodic solution from (b) and the solution to the IVP in (c). Choose a time interval so that you can clearly see the convergence of the IVP solution to the steady periodic solution.

3. Consider the same forced oscillator equation, except with a relatively small damping coefficient of $c = 0.2$ Unlike in the previous problem we will consider variable angular frequencies ω. This gives following differential equation:

$$x''(t) + 0.2x'(t) + 0.64x(t) = 2\cos(\omega t)$$

This system is a slightly damped forced oscillator, and we will investigate its practical resonance:

(a) Use the formula (21) in section 5.6, on page 384 of the textbook (page 355 in older version) to create a plot of the amplitude C of the steady periodic solution $x_{sp}(t)$ as a function of the driving angular frequency ω.

(b) Explain why the steady periodic amplitude peaks at a value near $\omega = 0.8$. Use calculus to find the exact value of ω that gives the maximum amplitude C.