Math 2250-010

Week 6 concepts and homework, due Wednesday February 12 by 5:00 p.m.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in. This 3.4-3.6 material will be represented on the Friday February 14 midterm exam.

(3.4 homework carried over from last week: matrix operations and algebra
3.4: 3, 5, 7, **10,** 13**,** **16**, 19, 27, 31, **32**, **34**, 39, **40**, 44.)

3.5: Matrix inverses
Formula for inverses of 2 by 2 matrices, and matrix algebra applications: 5, 7, 23,
w6.1: Let

$$A := \begin{bmatrix} 3 & 2 \\ 7 & 9 \end{bmatrix}$$

w6.1a) Find A^{-1} using the special (adjoint) formula for inverses of 2 by 2 matrices on page 191.
w6.1b) Find A^{-1} using the Gaussian elimination algorithm, where you reduce A augmented with the identity matrix. (Which do you prefer in this case, the method in a or the method in b?)
w6.1c) Use your formula for A^{-1} to solve the system

$$\begin{bmatrix} 3 & 2 \\ 7 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}.$$

w6.1d) Use A^{-1} to solve for the mystery matrix X in the following matrix equation. Check that your answer works!

$$X\begin{bmatrix} 3 & 2 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 13 & 13 \\ 7 & 9 \end{bmatrix}.$$

w6.1e) Use matrix algebra to solve for X. Verify that your answer works (with technology or by hand)! Hint: in order to factor out the matrix X (on the left), on the left side of the equation below, rewrite $2\,X$ as $2\,I\,X$, where I is the 2×2 identity matrix.

$$\begin{bmatrix} 1 & 2 \\ 7 & 7 \end{bmatrix} X + 2\,X = \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix}.$$

Gaussian elimination algorithm to deduce whether inverses exist, and to find them when they do: 3.5.9, 13, 21,
w6.2) Use the Gaussian elimination algorithm to determine that the matrix A below is not invertible, whereas the matrix B is. Use the algorithm that begins by augmenting a matrix with the identity matrix, in order to find the inverse matrix B^{-1}.

$$A := \begin{bmatrix} -1 & 1 & -4 \\ -1 & -1 & 2 \\ 4 & 1 & 1 \end{bmatrix} \qquad B := \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ -2 & 1 & -2 \end{bmatrix}$$

3.6 Determinants
Cofactor expansions: 3, **6.**
Combining cofactor expansions with elementary row operations to compute determinants: 11, 17.
The adjoint formula for matrix inverses 25, 33, *and Cramer's rule for finding individual components of the solution vector:* 21, 31.

w6.3a) Use Cramer's rule to re-solve for x and y in the linear system **w6.1c.**

w6.3b) Compute the determinants of the two matrices in **w6.2**, and verify that the determinant test correctly identifies the invertible matrix.

w6.3c) Use the adjoint formula to re-find B^{-1} in **w6.2**.

w6.3d) Use B^{-1} to solve the system

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ -2 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

w6.3e) Re-solve for the y -variable in **w6.3d)**, using Cramer's Rule.