Math 2250-010

Week 4 concepts and homework, due January 31.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

2.3: improved velocity-acceleration models:
constant, or constant plus linear drag forcing: **2, 3**, **9**, **10** (in **3** assume that the boat has only slowed to $25\ \frac{ft}{s}$ after *10 s* , rather than *20*. In **10** assume that the woman waits *25* seconds to open the parachute, rather than *20*.)
quadratic drag: **13, 14, 17, 19**
escape velocity: 25, 26. **Note:** 25, 26, are no longer required.

2.4-2.6: *numerical methods for approximating solutions to first order initial value problems. Your lab this week addresses these sections too.*
Note: 2.4.4, 2.5.4, 2.6.4 are no longer required.
2.4: 4: *Euler's method*
2.5: 4: *improved Euler*
2.6: 4*: Runge-Kutta*

w4.1) Runge-Kutta is based on Simpson's rule for numerical integration. Simpson's rule is based on the fact that for a subinterval of length $2h$, which by translation we may assume is the interval $-h \le x \le h$, the parabola $y = p(x)$ which passes through the points $(-h, y_0)$, $(0, y_1)$, (h, y_2) has integral

$$\int_{-h}^{h} p(x)\, dx = \frac{2h}{6} \cdot \left(y_0 + 4y_1 + y_2\right). \qquad (1)$$

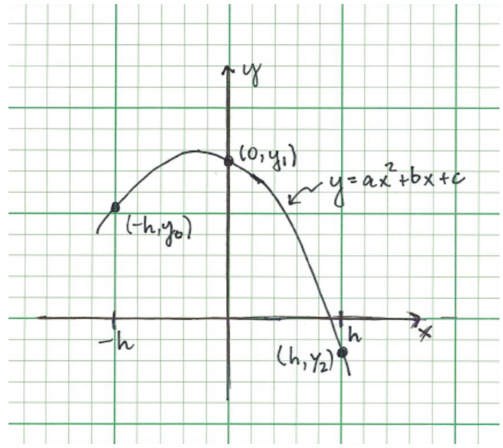
If we write the quadratic interpolant function $p(x)$ whose graph is this parabola as

$$p(x) = a x^2 + b x + c$$

with unknown parameters a, b, c then since we want $p(0) = y_1$ we solve $y_1 = p(0) = 0 + 0 + c$ to deduce that $c = y_1$.

w4.1a) Use the requirement that the graph of $p(x)$ is also to pass through the other two points, $(-h, y_0)$, (h, y_2) to express a, b in terms of h, y_0, y_1, y_2 .

w4.1b) Compute $\int_{-h}^{h} p(x)\, dx$ for these values of a, b, c and verify equation (1) above.



Remark: If you've forgotten, or if you never talked about Simpson's rule in your Calculus class, here's how it goes: In order to approximate the definite integral of $f(x)$ on the interval $[a, b]$, you subdivide $[a, b]$ into $2n = N$ subintervals of width $\Delta x = \frac{b-a}{2n} = h$. Label the $x-$ values $x_0 = a, x_1 = a + h, x_2 = a + 2h, \ldots x_{2n} = b$, with corresponding $y-$values $y_i = f(x_i)$, $i = 0, \ldots n$. On each successive pair of intervals use the stencil above, estimating the integral of f by the integral of the parabola. This yields the very accurate (for large enough n) Simpson's rule formula

$$\int_a^b f(x)\,\mathrm{d}x \approx \frac{2h}{6}\left(\left(y_0 + 4y_1 + y_2\right) + \left(y_2 + 4y_3 + y_4\right) + \ldots + \left(y_{2n-2} + 4y_{2n-1} + y_{2n}\right)\right),$$

i.e.

$$\int_a^b f(x)\,\mathrm{d}x \approx \frac{b-a}{6n}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 2y_{2n-2} + 4y_{2n-1} + y_{2n}\right).$$

http://en.wikipedia.org/wiki/Simpson's_rule