Math 2250-010
Mon Apr 7

Laplace transform:

- Using the unit step function to turn forcing on and off - Exercises 1b, 3 in Wednesday's notes.
- Impulse forcing ("delta functions") - Exercise 1 in Friday's notes.
- "Guess the resonance" game using convolution, Exercise 2 in Friday's notes.

Electrical circuits (today's notes):

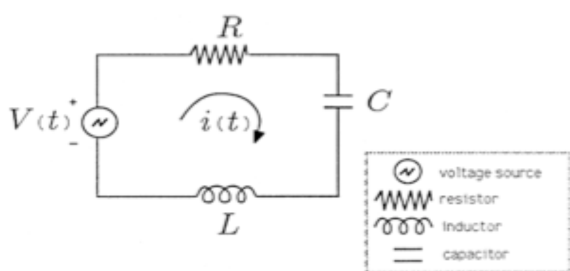
- Review the DE's for RLC circuits
- practical resonance in an RLC circuit, Exercise 1 in today's notes
- current surge in an RLC circuit, relates to lab problems.and do a problem related to your lab,

Exercise 2 in today's notes

We'll hold off on starting Chapter 7, systems of differential equations, until Wednesday.

Electrical RLC circuit review (originally in March 19 notes):

Practical resonance is usually bad in mechanical systems, but good in electrical circuits when signal amplification is a goal....recall from earlier in the course:



circuit element	voltage drop	units
inductor	$L\,I'(t)$	L Henries (H)
resistor	$R\,I(t)$	R Ohms (Ω)
capacitor	$\frac{1}{C}Q(t)$	C Farads (F)

http://cnx.org/content/m21475/latest/pic012.png

Kirchoff's Law: The sum of the voltage drops around any closed circuit loop equals the applied voltage $V(t)$ (volts). If the applied voltage is sinusoidal this leads to

For $Q(t)$: $$L\,Q''(t) + R\,Q'(t) + \frac{1}{C}Q(t) = V(t) = E_0 \sin(\omega\,t)$$

For $I(t)$: $$L\,I''(t) + R\,I'(t) + \frac{1}{C}I(t) = V'(t) = E_0\,\omega \cos(\omega\,t)\ .$$

We can transcribe the work on steady periodic solutions to forced mechanical systems that we dervied earlier, and apply it directly to forced RLC circuits. Recall that for

$$m\,x'' + c\,x' + k\,x = F_0 \cos(\omega t)$$

we found the steady periodic solution

$$x_{sp}(t) = A\cos(\omega t) + B\sin(\omega t) = C\cos(\omega t - \alpha)$$

with

$$C = \frac{F_0}{\sqrt{\left(k - m\,\omega^2\right)^2 + c^2\omega^2}}$$

$$\cos(\alpha) = \frac{k - m\,\omega^2}{\sqrt{\left(k - m\,\omega^2\right)^2 + c^2\omega^2}}$$

$$\sin(\alpha) = \frac{c\,\omega}{\sqrt{\left(k - m\,\omega^2\right)^2 + c^2\omega^2}}\,.$$

Exercise 1) We know that the general solution for $I(t)$ is

$$I(t) = I_{sp}(t) + I_{tr}(t)\,.$$

Transcribe the results for forced mechanical oscillations and use some algebra to deduce that for

$$I_{sp}(t) = I_0\cos(\omega t - \alpha) \qquad \left(= I_0\sin(\omega t - \gamma)\,,\ \ \gamma = \alpha - \frac{\pi}{2}\right)$$

the amplitude of the steady periodic solution is given by

$$I_0 = I_0(\omega) = \frac{E_0}{\sqrt{\left(\frac{1}{C\omega} - L\,\omega\right)^2 + R^2}}\,.$$

The denominator $\sqrt{\left(\frac{1}{C\omega} - L\,\omega\right)^2 + R^2}$ of $\mathrm{I}_0(\omega)$ is called the impedance $Z(\omega)$ of the circuit (because the larger the impedance, the smaller the amplitude of the steady-periodic current that flows through the circuit) . Notice that for fixed resistance, the impedance is minimized and the steady periodic current amplitude is maximized when $\frac{1}{C\,\omega} = L\,\omega$, i.e.

$$C = \frac{1}{L\,\omega^2} \quad \text{if } L \text{ is fixed and } C \text{ is adjustable (old radios).}$$

$$L = \frac{1}{C\,\omega^2} \quad \text{if } C \text{ is fixed and } L \text{ is adjustable}$$

Both L and C are adjusted in this M.I.T. lab demonstration:

http://www.youtube.com/watch?v=ZYgFuUl9_Vs.

Exercise 2) The following example is qualitatively like what could happen if an electrical grid is subjected to a growing demand on a hot day, when a lot of people start getting home from work at 5:00 and proceed to turn on their air conditioners. (See also problem 2 in this week's lab.) Solve this initial value problem for current $I(t)$, and discuss the behavior of the solution:

$$I'' + 4\,I' + 104\,I = 2\,u(t-5)$$
$$I(0) = 0$$
$$I'(0) = 0$$