Math 2250-010
Wednesday, April 23
Course review

Final exam: Tuesday April 29, 8:00 a.m. -10:00 a.m. (I will let you work until 10:15). This is the official University time and location - our lecture room SW 137 (here). As usual the exam is closed book and closed note, and the only sort of calculator that is allowed is a simple scientific one. You will be provided a Laplace Transform table. The algebra and math on the exam should all be doable by hand.

Review of previous final exam: Time and date TBD in class Wednesday, and then I'll find a location.

The final exam will be comprehensive, but weighted to more recent material. Rough percentage ranges per chapter are below – these percentage ranges add up to more than 100% because many topics span several chapters. Also consult the "course learning objectives" in our syllabus.

Chapters

- 1-2: 10-20% first order DEs
- 3-4: 20-30% matrix algebra and vector spaces
- 5, EP3.7: 15-30% linear differential equations and applications
- 6.1-6.2: 15-30% eigenvalues and eigenvectors, including complex case
- 7.1-7.4: 20-40% linear systems of differential equations and applications
- 9.1-9.4: 15-20% non-linear autonomous systems of DEs and applications
- 10.4-10.5, EP 7.6: 15-30% Laplace transform

On the next page is a more detailed list of the topics we've investigated this semester. They are more inter-related than you may have realized at the time, so let's discuss the connections. Then we'll work an extended problem that lets us highlight these connections and review perhaps 70% of the key ideas in this course.

1-2: first order DEs

- slope fields, Euler approximation
- phase diagrams for autonomous DEs
  - equilibrium solutions
  - stability
- existence-uniqueness thm for IVPs
- methods:
  - separable
  - linear
- applications
  - populations
  - velocity-acceleration models
  - input-output models

3-4 matrix algebra and vector spaces

- linear systems and matrices
- reduced row echelon form
- matrix and vector algebra
  - manipulating and solving matrix-vector equations for unknown vectors or matrices.
- matrix inverses
- determinants
- vector space concepts
  - vector spaces and subspaces
  - linear combinations
  - linear dependence/independence
  - span
  - basis and dimension
  - linear transformations
    - aka superposition
    - fundamental theorem for solution space to L(y)=f when L is linear

5 Linear differential equations

- IVP existence and uniqueness
- Linear DEs
  - Homogeneous solution space, its dimension, and why
  - superposition, $\underline{x}(t)= \underline{x}_P+ \underline{x}_H$
- Constant coefficient linear DEs
  - $\underline{x}_H$ via characteristic polynomial
    - Euler's formula, complex roots
  - $\underline{x}_P$ via undetermined coefficients
  - solving IVPs
- applications:
  - mechanical configurations
    - unforced: undamped and damped
      - cos and sin addition angle formulas and amplitude-phase form
    - forced undamped: beating, resonance
    - forced damped: $\underline{\mathbf{x}}_{sp}+ \underline{\mathbf{x}}_{tr}$, practical resonance
    - RLC circuits
    - Using conservation of total energy (=KE+PE) to derive equations of motion, especially for mass-spring and pendulum

6.1-6.1 eigenvalues, eigenvectors (eigenspaces), diagonalizable matrices; real+complex eigendata.

7.1-7.4 linear systems of DEs

- first order systems of DEs and tangent vector fields.
- existence-uniqueness thm for first order IVPs
- superposition, $\underline{\mathbf{x}}= \underline{\mathbf{x}}_P+ \underline{\mathbf{x}}_H$
- dimension of solution space for $\underline{\mathbf{x}}_H$.
- conversion of DE IVPs or systems to first order system IVPs.
- Constant coefficient systems and methods:
  - $\underline{\mathbf{x}}'(t)= A\underline{\mathbf{x}}$
  - $\underline{\mathbf{x}}'(t)= A\underline{\mathbf{x}}+\underline{\mathbf{f}}(t)$
  - $\underline{\mathbf{x}}''(t)= A\underline{\mathbf{x}}$ (from conservative systems)
  - $\underline{\mathbf{x}}''(t)= A\underline{\mathbf{x}}+\underline{\mathbf{f}}(t)$
- applications: phase portrait interpretation of unforced oscillation problems; input-output modeling; force and unforced mass-spring systems.

9.1-9.4 non-linear systems of differential equations

- autonomous systems of first order DEs
  - equilibrium solutions
  - stability
  - phase portraits
  - linearization near equilibria, stability analysis, further classification and qualitative sketching.
  - Applications to interacting populations and non-linear mechanical configurations, esp. pendulum

10.1-10.5, EP7.6: Laplace transform

- definition, for direct computation
- using table for Laplace and inverse Laplace transforms … including for topics before/after the second midterm, i.e. on/off and impulse, forcing, convolution solutions
- Solving linear DE (or system of DE) IVPs with Laplace transform.

We can illustrate many ideas in this course, and how they are tied together by studying the following two differential equations in as many ways as we can think of.

$x''(t) + 5\,x'(t) + 4\,x(t) = 0$

$x''(t) + 5\,x'(t) + 4\,x(t) = 3\cos(2t)$