Name_________________________

Student I.D.___________________

Math 2250-4
Quiz 9 SOLUTIONS
March 22, 2013

1a) Find a particular solution to the undamped forced oscillator differential equation for $x(t)$ given by

$$x''(t) + 4\,x(t) = 10\cos(3\,t).$$

(5 points)

Undetermined coefficients would ordinarily say we should try $x_P(t) = x(t) = A\cos(3\,t) + B\sin(3\,t)$ *but since the left side of the differential equation only has even derivatives we can try*

$$x(t) = A\cos(3\,t)$$
$$x'(t) = -3\,A\sin(3\,t)$$
$$x''(t) = -9\,A\cos(3\,t)$$

so for this guess,

$$x''(t) + 4\,x(t) = -9\,A\cos(3\,t) + 4\,A\cos(3\,t) = -5\,A\cos(3\,t)$$

In order for this to equal $10\cos(3\,t)$ *we pick* $A = -2$ *and get*

$$x_P(t) = -2\cos(3\,t).$$

1b) What is the general solution to the differential equation above?

(2 points)

$$x = x_P + x_H.$$

The homogeneous solution $x_H(t)$ *solves the undamped unforced harmonic oscillator equation*

$$x''(t) + 4\,x(t) = 0$$

which has $\omega_0 = 2$ *and solution*

$$x_H(t) = c_1\cos(2\,t) + c_2\sin(2\,t).$$

<u>Note:</u> you should learn to recognize the equation and solution above. It will save you time vs. going through the characteristic polynomial one more time:

$$p(r) = r^2 + 4 = 0 \Rightarrow r^2 = -4 \Rightarrow r = \pm 2\,i \Rightarrow$$
$$\text{complex solutions } e^{\pm 2\,i\,t} \Rightarrow \text{ real solutions } \cos(2\,t), \sin(2\,t).$$

In either case, we deduce the general solution

$$x = x_P + x_H = -2\cos(3\,t) + c_1\cos(2\,t) + c_2\sin(2\,t).$$

2a) What form would the undetermined coefficients particular solution take, for the forced oscillator equation

$$x''(t) + 4\,x(t) = 10\cos(2\,t)\ ?$$

(You don't need to find the precise particular solution.)

(2 points)

Since $\cos(2\,t)$ *solves the homogeneous DE and corresponds to the characteristic polynomial roots* *r* $=\pm 2i$ *we multiply the standard undetermined coefficients guess by the variable "t":*

$$x_P(t) = t\,(A\cos(2t) + B\sin(2t)).$$

(It turns out that we didn't need the "cos" term in the x_P *guess, although we only figured that out later.)*

2b) What is the name of the phenomenon that solutions to this differential equation will exhibit?

(1 point)

$\omega = \omega_0 \Rightarrow$ *(pure) resonance.*