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Student I.D.__________________

Math 2250-4
Quiz 7 SOLUTIONS
March 1, 2013

1a) Consider the differential equation for $y(x)$

$$y''(x) - 5\,y'(x) + 6\,y(x) = 0\,.$$

Find the general solution to this homogeneous differential equation. Hint: the solution space has a basis consisting of exponential functions.

(5 points)

If we try $y(x) = e^{r\,x}$ we compute

$$L\left(e^{r\,x}\right) = r^2\,e^{r\,x} - 5\,r\,e^{r\,x} + 6\,e^{r\,x} = e^{r\,x}\left(r^2 - 5\,r + 6\right).$$

Since we want $L\left(e^{r\,x}\right) = 0$ *the exponential growth rates r must be roots of the characteristic polynomial, i. e they must solve*

$$0 = r^2 - 5\,r + 6 = (r-3)(r-2)\,.$$

Since the roots are $r = 2, 3$, $y_1(x) = e^{2\,x}$, $y_2(x) = e^{3\,x}$ *solve the DE, and the general homogeneous solution is*

$$y_H(x) = c_1 e^{3\,x} + c_2 e^{2\,x}.$$

1b) Verify that $y(x) = \mathrm{e}^{4\,x}$ is a solution to

$$y''(x) - 5\,y'(x) + 6\,y(x) = 2\,\mathrm{e}^{4\,x}.$$

(1 points)

For $y(x) = \mathrm{e}^{4\,x}$,

$$y''(x) - 5\,y'(x) + 6\,y(x) = 16\,\mathrm{e}^{4\,x} - 20\,\mathrm{e}^{4\,x} + 6\,\mathrm{e}^{4\,x} = 2\,\mathrm{e}^{4\,x}.$$

1c) Use your work from a, b to deduce the general solution to the non-homogeneous DE in b, and use this general solution to solve the initial value problem

$$y''(x) - 5\,y'(x) + 6\,y(x) = 2\,\mathrm{e}^{4\,x}$$
$$y(0) = 0$$
$$y'(0) = 0.$$

(4 points)

The general solution is

$$y(x) = y_P(x) + y_H(x) = \mathrm{e}^{4\,x} + c_1 e^{3\,x} + c_2 e^{2\,x}.$$

So

$$y'(x) = y_P{}'(x) + y_H{}'(x) = 4\,\mathrm{e}^{4\,x} + 3\,c_1 e^{3\,x} + 2\,c_2 e^{2\,x}\,.$$

To solve the IVP we set x = 0 and solve for the free parameters:

$$y(0) = 0 = 1 + c_1 + c_2$$
$$y'(0) = 0 = 4 + 3\,c_1 + 2\,c_2.$$
$$c_1 + c_2 = -1$$
$$3\,c_1 + 2\,c_2 = -4$$

$$\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ -4 \end{bmatrix} = - \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

$$y(x) = \mathrm{e}^{4x} - 2e^{3x} + e^{2x}.$$