Name________________________
Student I.D.___________________

Math 2250-4
Quiz 6 SOLUTIONS
February 22, 2013

1a) Define the <u>span</u> of a collection of vectors $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$.

(2 points)

• *The <u>span</u> of a collection of vectors* $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$ *is the collection of all linear combinations of* $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$.

• *You can say this equivalently using set notation:*

$$span\{\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n\} = \{c_1\underline{v}_1 + c_2\underline{v}_2 + \ldots + c_n\underline{v}_n, \text{ such that } c_1, c_2, \ldots c_n \in \mathbb{R}\}$$

(You only need to give one correct formulation of the definition to get credit, of course.)

1b) Define what it means for vectors $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$ to be <u>linearly independent.</u>

(2 points)

• $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$ *are <u>linearly independent</u> means that the only linear combination of them that equals the zero vector is the one for which all the linear combination coefficients are zero.*

• *You can say this using mathematical notation:*

$$c_1\underline{v}_1 + c_2\underline{v}_2 + \ldots + c_n\underline{v}_n = \underline{\mathbf{0}} \Rightarrow c_1 = c_2 = \ldots = c_n = 0.$$

• *an equivalent definition is that none of the vectors* $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$ *are linear combinations of the others.*

• *another equivalent definition (which is the real reason we like linearly independent collections of vectors) is that for each vector* $\underline{v}$ *in the span of* $\underline{v}_1, \underline{v}_2, \ldots \underline{v}_n$ *, i.e. for each* $\underline{v} = c_1\underline{v}_1 + c_2\underline{v}_2 + \ldots + c_n\underline{v}_n$, *the linear combination coefficients* $c_1, c_2, \ldots c_n$ *are unique.*

2) Consider the following four vectors in $\mathbb{R}^4$:

$$\underline{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -4 \\ 2 \end{bmatrix}, \underline{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \underline{v}_3 = \begin{bmatrix} 1 \\ 4 \\ 6 \\ 2 \end{bmatrix}, \underline{v}_4 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}.$$

Below is the matrix which has these four vectors as columns, in the same order as above, and the reduced row echelon form of that matrix. Use this information to answer the three questions which follow.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 4 & 1 \\ -4 & 1 & 6 & 2 \\ 2 & 2 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

2a) Are the four vectors $\underline{v}_1, \underline{v}_2, \underline{v}_3, \underline{v}_4$ a basis for $\mathbb{R}^4$? (In other words, is it true that they span $\mathbb{R}^4$ and are linearly independent?) Briefly explain your answer.

(2 points)

No, they are not a basis for $\mathbb{R}^4$. *They are not linearly independent. Also, they do not span* $\mathbb{R}^4$. *(We know these two facts because the reduced row echelon form of the matrix above is not the identity.)*

2b) Express $\underline{v}_3$ as a linear combination of the other three vectors $\underline{v}_1, \underline{v}_2, \underline{v}_4$. (Hint: One of the linear combination coefficients is zero.)

(2 points)

Reading the column dependencies for the original matrix off of the column dependencies of the reduced matrix, we see that

$$\underline{\boldsymbol{v}}_3 = -\underline{\boldsymbol{v}}_1 + 2\,\underline{\boldsymbol{v}}_2 .$$

$$\text{(check: } \begin{bmatrix} 1 \\ 4 \\ 6 \\ 2 \end{bmatrix} = -\begin{bmatrix} 1 \\ 0 \\ -4 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}. \text{ yup)}$$

2c) Are the vectors $\underline{\boldsymbol{v}}_1$, $\underline{\boldsymbol{v}}_2$, $\underline{\boldsymbol{v}}_4$ linearly independent? Explain.

(2 points)

Yes. Because the corresponding columns of the reduced matrix are linearly independent.