Name________________________
Student I.D.___________________

Math 2250-4
Quiz 6
February 22, 2013

1a) Define the span of a collection of vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_n$.

(2 points)

1b) Define what it means for vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \ldots \underline{\boldsymbol{v}}_n$ to be linearly independent.

(2 points)

2) Consider the following four vectors in $\mathbb{R}^4$:

$$\underline{\boldsymbol{v}}_1 = \begin{bmatrix} 1 \\ 0 \\ -4 \\ 2 \end{bmatrix}, \quad \underline{\boldsymbol{v}}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \quad \underline{\boldsymbol{v}}_3 = \begin{bmatrix} 1 \\ 4 \\ 6 \\ 2 \end{bmatrix}, \quad \underline{\boldsymbol{v}}_4 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}.$$

Below is the matrix which has these four vectors as columns, in the same order as above, and the reduced row echelon form of that matrix. Use this information to answer the three questions which follow.

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 4 & 1 \\ -4 & 1 & 6 & 2 \\ 2 & 2 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

2a) Are the four vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \underline{\boldsymbol{v}}_3, \underline{\boldsymbol{v}}_4$ a basis for $\mathbb{R}^4$? (In other words, is it true that they span $\mathbb{R}^4$ and are linearly independent?) Briefly explain your answer.

(2 points)

2b) Express $\underline{\boldsymbol{v}}_3$ as a linear combination of the other three vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \underline{\boldsymbol{v}}_4$. (Hint: One of the linear combination coefficients is zero.)

(2 points)

2c) Are the vectors $\underline{\boldsymbol{v}}_1, \underline{\boldsymbol{v}}_2, \underline{\boldsymbol{v}}_4$ linearly independent? Explain.

(2 points)