Name________________________
Student I.D.___________________

Math 2250-4
Quiz 5 SOLUTIONS
February 8, 2013

1a) Consider the following system of equations

$$2x + y + 3z = 1$$
$$3x + 3z = 2$$
$$-x - 2y - 3z = 0$$

Exhibit the augmented matrix corresponding to this system, compute its reduced row echelon form, and find the solution set to the system.

(5 points)

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 3 & 0 & 3 & 2 \\ -1 & -2 & -3 & 0 \end{bmatrix}$$

$swap\ R_1, -R_3$:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 3 & 0 & 3 & 2 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

$-3R_3 + R_2 \rightarrow R_2,\ -2R_1 + R_3 \rightarrow R_3$:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -6 & -6 & 2 \\ 0 & -3 & -3 & 1 \end{bmatrix}$$

$-R_3 \rightarrow R_2,\ R_2 \rightarrow R_3$:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 3 & 3 & -1 \\ 0 & -6 & -6 & 2 \end{bmatrix}$$

$2R_2 + R_3 \rightarrow R_3$:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 3 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\frac{1}{3}R_2 \rightarrow R_2$:

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 1 & -\frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-2R_2 + R_1 \to R_1$:

$$\begin{bmatrix} 1 & 0 & 1 & \frac{2}{3} \\ 0 & 1 & 1 & -\frac{1}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

backsolving from the reduced row echelon form of the augmented matrix we have $z = t \in \mathbb{R}$, $y = -\frac{1}{3} - t$, $x = \frac{2}{3} - t$. *In vector form the explicit solution is*

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{2}{3} - t \\ -\frac{1}{3} - t \\ t \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}.$$

1b) Interpreting the solution set of each single equation above as a plane in $\mathbb{R}^3$, what geometric configuration corresponds to the solution set of the system of 3 equations above?

(2 points)

This is three planes intersecting in a line. This line goes through the point $\left[\frac{2}{3}, -\frac{1}{3}, 0\right]^T$ *and its direction is parallel to* $[-1, -1, 1]^T$.

2) Consider the matrix equation

$$\begin{bmatrix} 1 & -2 & 3 & 4 \\ 5 & -2 & 7 & 3 \\ 3 & 2 & 1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

with $\underline{b} \neq \mathbf{0}$. Without trying to find the solution set explicitly, explain which of the following three outcomes are possible for the solution set, just based on the number of equations, the number of unknowns, and the right hand side: (a) no solutions; (b) exactly one solution; (c) infinitely many solutions.

(3 points)

There could be no solutions (a). This will happen if one of the rows of the reduced row echelon form of the augmented matrix is $[0\ 0\ 0\ 0 \mid 1]$. *If the system is consistent then there will still be at least one column* *without a leading 1 in the reduced row echelon form of the coefficient matrix (since there can be at most 3 leading 1's, but there are 4 columns), so there will be infinitely many solutions (c). So case (b) cannot occur under any circumstances.*