Name_____

Student I.D._____

Math 2250-4
Quiz 4 SOLUTIONS
February 1, 2013

1) Consider the following linear drag initial value problem:

$$\frac{dv}{dt} = -20 - 2v.$$
$$v(0) = 0.$$

1a) Use a phase diagram to determine the limiting velocity $\lim_{t \to \infty} v(t)$ for the solution to this IVP.

(2 points)

since $-20 - 2v = -2(v + 10)$ *the equilibrium solution is* $v = -10$ *and the phase diagram is*

$$\rightarrow \rightarrow \rightarrow -10 \leftarrow \leftarrow \leftarrow \leftarrow$$

Thus, no matter the value of v_0, $\lim_{t \to \infty} v(t) = -10$.

1b) Solve the initial value problem above. (Your solution should be consistent with the correct answer to part (a) above.)

(6 points)

May use linear or separable algorithms. For linear:

$$v'(t) + 2v(t) = -20$$
$$e^{2t}(v'(t) + 2v(t)) = -20e^{2t}$$
$$e^{2t}v(t) = \int -20e^{2t}dt = -10e^{2t} + C$$
$$\Rightarrow v(t) = -10 + Ce^{-2t}.$$
$$v(0) = 0 \Rightarrow C = 10 \Rightarrow v(t) = -10e^{-2t} - 10.$$

1c) Use Newton's Law to convert the following information below into a differential equation initial value problem - your IVP should end up being equivalent to the IVP at the top of this page, if you keep track of your units correctly.

A cannister is suspended underwater, at the surface of a deep body of water. At time $t = 0$ the cannister is released, to descend into the depths. The cannister weighs 320 pounds on dry land but because of its volume it is also subject to a buoancy force of 120 pounds when underwater. In addition, while descending the cannister is subject to a drag force of 20 pounds for each foot per second of speed. Specifying that "up" is the positive direction, derive the IVP for the cannister's velocity after release. (Hint: recall that a dry-land weight of 32 pounds = mg corresponds to a mass of 1 slug in the English system.)

(2 points)

By Newton's second law,

$$m\,v'(t) = \text{net forces} = F_{gravity} + F_{buoancy} + F_{drag} = -mg + 120 - 20v.$$

Since the dry-land weight is 320 pounds, the mass is 10 slugs, so the DE is

$$10\,v'(t) = -320 + 120 - 20v = -200 - 20v$$

which simplifies to

$$v'(t) = -20 - 2v.$$

Since the cannister starts out at rest the initial condition is $v(0) = 0$.