Name________________________

Student I.D.____________________

Math 2250-4
Quiz 11 SOLUTIONS
April 12, 2013

1a) Use the methods we've been discussing to find the general solution to the system of differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

(8 points)

$$\begin{vmatrix} 0-\lambda & 1 \\ -8 & -6-\lambda \end{vmatrix} = \lambda(\lambda+6)+8 = \lambda^2+6\lambda+8 = (\lambda+4)(\lambda+2)$$

$\lambda = -4$ *the homogeneous system is*

$$\left[\begin{array}{cc|c} 4 & 1 & 0 \\ -8 & -2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 4 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \underline{v} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$\lambda = -2$ *the homogeneous system is*

$$\left[\begin{array}{cc|c} 2 & 1 & 0 \\ -8 & -4 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \underline{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

Thus the general solution is

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 e^{-4t} \begin{bmatrix} 1 \\ -4 \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

1b) For any solution $[x(t), y(t)]^T$ to the first order system of DEs above, what is the second order differential equation satisfied by $x(t)$?

(2 points)

The first order system reads

$$x'(t) = y$$
$$y'(t) = -8x - 6y.$$

Thus $x'' = y' = -8x - 6y = -8x - 6x'$. *We can also write this DE in the usual form for an unforced undamped mechanical or electrical system*

$$x'' + 6x' + 8x = 0.$$

(And we continued this discussion in class after the quiz, remarking once again on the correspondence between Chapter 5 and Chapter 7, in the cases where first order systems of DE's arise from converting n^{th} order linear DE's to systems of n first order linear DE's.)