Math 2250
Second Software (Maple/Matlab/Mathematica/other) Assignment
Numerical approximation for solutions to Differential Equations
due February 4 at midnight

If you choose to do this project using Maple, begin by downloading this document and opening it from Maple. A shortcut is to use the "open URL" option in the "File" menu item.
http://www.math.utah.edu/~korevaar/2250spring13/maplehw2.mw

Also recall the list of commands you might find helpful for this course, posted at
http://www.math.utah.edu/~korevaar/2250spring13/maplecommands.mw

In this project you will be using some small "for loops" with "conditionals" to do the iterations. You might also investigate the examples in the help documentation:

> ? *for;*

> ? *conditional*

Wikipedia has good explanations of these basic programming ideas - just google "for loop wikipedia" or "conditional (programming) wikipedia". You will be using the Euler and Runge-Kutta numerical techniques. Sections 2.4 and 2.6 contain pseudocode for these algorithms. There are Matlab and TI-85 implementations at the ends of these sections. There are Maple implementations illustrating these algorithms in the Maple lecture notes for Math 2250-1 last fall September 10. We will discuss a version of these notes on Tuesday January 29.
http://www.math.utah.edu/~korevaar/2250fall12/sept10.mw

If you are proceding in Maple, you will create a Maple document containing a mixture of text and mathematics, in which you answer various mathematical questions. If you are proceding in Matlab, please follow the directions posted elsewhere. You will submit your document(s) as specified by your instructor.

Important directions for submission for Math 2250-4 Korevaar: Create your document so that we can regenerate your answers by using the Edit/Execute/Worksheet menu option . Before you submit your .mw solution file on CANVAS, remove all Maple output using the menu option Edit/Remove Output/From Worksheet. If you're worried that your file may become corrupted during submission and we won't be able to regenerate your answers, you may also submit a .pdf or .ps printout which includes all the output, in addition to the .mw file.

Problem 0: Create a new document, and follow the same formatting directions as in the first project. Remember to only do mathematical computations inside of Math execution groups, in your work below.

Now, consider the initial value problem
$$v'(t) = 32 - 0.2\, v$$
$$v(0) = 0$$

You solved this initial value problem in the first Maple project - the solution is
$$v(t) = 160 - 160e^{-0.2\,t}\,.$$

This problem and its solution could be modeling a linear drag problem for a package dropped out of a helicopter at time $t = 0$. In this model we measure positive distance in the downwards direction; the acceleration of gravity is $32 \ \frac{ft}{s^2}$, and the drag of the parachute adds the deceleration term $-0.2 \, v \ \frac{ft}{s^2}$. In the first project you figured out what percentage of the terminal velocity of $160 \ \frac{ft}{s}$ is obtained after 2 and 5 seconds.

__1)__ Use Euler's method to approximate the solution to the initial value problem on the interval $0 \le t \le 5$, as follows:

__1a)__ Use step size $h = 0.5$. Write your "for" loop so that Maple only prints out the approximate values at $t = 1, 2, 3, 4, 5$. What is the relative error, in percent, of your numerical estimates as compared to the exact solution values, at $t = 2, 5$? (Recall that relative error is the error divided by the exact value.)
__1b)__ Now use step size $h = 0.05$, display at the same t-values and answer the same questions about the relative accuracy as in part (1a)
__1c)__ Now use step size $h = 0.005$ and display at the same t-values and answer the same questions about the relative accuracy as in parts (1a) and (1b).

__2)__ Use Runge-Kutta for the same initial value problem and the same interval. Use step size $h = 1$. Answer the same questions as in (1a). How does the accuracy of Runge-Kutta with step size $h = 1$ compare to the accuracy of Euler with the three step sizes you used in (1)?

__3)__ Now we'll drop the same package, but with a parachute attached. The wind resistance provided by a parachute is not really a linear deceleration. Suppose that for a certain type of parachute and the package in the earlier discussion, the acceleration equation is more effectively modeled by
$$v'(t) = 32 - 0.3 \, v - 0.1 \, v^{1.7} \, .$$
$$v(0) = 0 \, .$$

__3a)__ What is the terminal velocity in this model? Notice that the "slope function" $32 - 0.3 \, v - 0.1 \, v^{1.7}$ is decreasing function of v for $v > 0$, that its value is 32 when $v = 0$, and that its limit as $v \to \infty$ is $-\infty$. Thus the slope function has exactly one root and you can find it with the Maple (algebraic)"solve" command. Don't expect a nice integer value for the terminal velocity. Since the modified drag function increases the drag from the previous model (because of the parachute), the new terminal velocity should be smaller than it was originally.

__3b)__ This is a differential equation which does NOT have an elementary solution. Use Runge-Kutta with step sizes $h = 0.2$ and $h = 0.02$ to estimate the solution values at $t = 2$ and $t = 5$ seconds. Based on a comparison of the estimated values at these two choices of h, how confident are you either one (or both) of them provides a good estimate for the actual values $v(2)$ and $v(5)$? Explain your reasoning.

__3c)__ Based on your work in part (3b), what percent of the terminal velocity is attained at $t = 2$ seconds and at $t = 5$ seconds?