Math 2250-4

Week 6 concepts and homework, due Wednesday February 13.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems. (There won't be a Friday quiz on February 15, but the 3.5-3.6 material will be represented on the Thursday February 14 midterm.)

3.5: Matrix inverses

Formula for inverses of 2 by 2 matrices, and matrix algebra applications: 5, 7, 23,

w6.1: Let

$$A := \begin{bmatrix} 2 & 3 \\ 7 & 4 \end{bmatrix}$$

w6.1a) Find A^{-1} using the special (adjoint) formula for inverses of 2 by 2 matrices on page 191.

w6.1b) Find A^{-1} using the Gaussian elimination algorithm, where you reduce A augmented with the identity matrix. (Which do you prefer in this case, the method in a or the method in b?)

w6.1c) Use your formula for A^{-1} to solve the system

$$\begin{bmatrix} 2 & 3 \\ 7 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 23 \end{bmatrix}.$$

w6.1d) Use A^{-1} to solve for the mystery matrix X in the following matrix equation. Check that your answer works (with technology or by hand)!

$$X \begin{bmatrix} 2 & 3 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 13 & 26 \\ 7 & 4 \\ 2 & 3 \end{bmatrix}.$$

w6.1e) Use matrix algebra and the inverse of a certain different matrix to solve for X. Verify that your answer works (with technology or by hand)!

$$X \begin{bmatrix} 2 & 3 \\ 7 & 4 \end{bmatrix} + 2\,X = \begin{bmatrix} 0 & 3 \\ 3 & 6 \end{bmatrix}.$$

Gaussian elimination algorithm to deduce whether inverses exist, and to find them when they do: 9, 13, 21,

w6.2) Use the Gaussian elimination algorithm to determine that the matrix A below is not invertible, whereas the matrix B is. Use the algorithm that begins by augmenting a matrix with the identity matrix, in order to find the inverse matrix B^{-1}.

$$A := \begin{bmatrix} -2 & 1 & -4 \\ 0 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix} \qquad B := \begin{bmatrix} -2 & 2 & -1 \\ 1 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$$

Technology when appropriate:
Automated solutions problem 2 page 201. The gold coins in a bag problem. Convert the word problem to a matrix problem, and use the inverse matrix to find the solution. Use your favorite software, internet site, or graphing calculator. (The textbook commands for Maple on page 200 are out of date, although they should still work. I'd recommend using the commands shown in the Maple Commands file on our 2250-4 homework page, and as we've been using in class.) Show work, and indicate at which stage you resorted to technology.

3.6 Determinants
Cofactor expansions: 3, **6.**
Combining cofactor expansions with elementary row operations to compute determinants: 11, 17.
The adjoint formula for matrix inverses 25, 33, *and Cramer's rule for finding individual components of the solution vector:* 21, 31.

w6.3a) Use Cramer's rule to re-solve for x and y in the linear system **w6.1b.**

w6.3b) Compute the determinants of the two matrices in **w6.2**, and verify that the determinant test correctly identifies the invertible matrix.

w6.3c) Use the adjoint formula to re-find B^{-1} in **w6.2**.

w6.3d) Use B^{-1} to solve the system

$$\begin{bmatrix} -2 & 2 & -1 \\ 1 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

w6.3e) Re-solve for the y -variable in **w6.3d)**, using Cramer's Rule.