

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

3.1-3.3: linear systems of (algebraic) equations; augmented matrices and Gaussian elimination to compute reduced row-echelon form. Explicitly specifying the solution space by backsolving the reduced row echelon form of the augmented matrix. Depending on your previous experience and background, you may need to practice this algebra a lot, or just a little. So, I've included a fairly large number of good practice problems in addition to the ones you'll hand in.

3.1: 1, 4, 6, 11, 16, 17, 19, 23, 24, 27, 28, 29, 32, 33, 34

3.2: 7, 8, 9, 13, 17, 20 (backsolve from row echelon form), 29, 30;

3.3: 13, 17, 20, 33, 34;

3.4: matrix operations and algebra

3.4: 3, 5, 7, 10, 13, 16, 19, 27, 31, 32, 34, 39, 40, 44.

w5.1 Consider the augmented matrix for 3.2.20 above:

$$\left[\begin{array}{ccccc|c} 2 & 4 & -1 & -2 & 2 & 6 \\ 1 & 3 & 2 & -7 & 3 & 9 \\ 5 & 8 & -7 & 6 & 1 & 4 \end{array} \right].$$

Continue your work from **3.2.20** to compute the reduced row echelon form of this augmented matrix, and verify that you recover the same explicit solutions as you did in that problem, when you backsolved from a row echelon form.

w5.2 The matrix A in this problem is like the one in **3.3.20**, except the 5th entry in the second row is a 2, instead of a 47.

a) Use technology to compute the reduced row echelon form of the matrix A below:

$$A := \left[\begin{array}{ccccc} 3 & 6 & 1 & 7 & 13 \\ 5 & 10 & 8 & 18 & 2 \\ 2 & 4 & 5 & 9 & 26 \end{array} \right]$$

b) Use the reduced row echelon form of the matrix A to find the general solution to the homogeneous linear system $A\mathbf{x} = \mathbf{0}$, i.e.

$$\begin{aligned} 3x_1 + 6x_2 + x_3 + 7x_4 + 13x_5 &= 0 \\ 5x_1 + 10x_2 + 8x_3 + 18x_4 + 2x_5 &= 0 \\ 2x_1 + 4x_2 + 5x_3 + 9x_4 + 26x_5 &= 0 \end{aligned}$$

(Hint: the augmented matrix would have a final (sixth) column of zeros appended to it. When you reduce, that column will stay zero.)

c) Check your answer to **b** with technology. Hand in evidence.

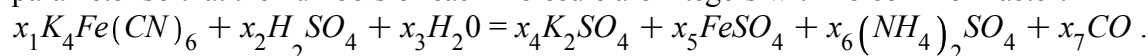
d) Since elementary row operations only change column entries using entries from the same column, you can use the reduced row echelon form of a single matrix to answer various linear system problems. For example, focus on the first four columns of the matrix A above, and its reduced row echelon form. Pretend there is an augmenting vertical line between the 3rd and 4th column, in order to read off the general solution to

$$\begin{aligned}3x_1 + 6x_2 + x_3 &= 7 \\5x_1 + 10x_2 + 8x_3 &= 18 \\2x_1 + 4x_2 + 5x_3 &= 9.\end{aligned}$$

e) Without doing any additional reduced row echelon form computations explain why the system below has no solutions:

$$\begin{aligned}3x_1 + 6x_2 + x_3 + 7x_4 &= 13 \\5x_1 + 10x_2 + 8x_3 + 18x_4 &= 2 \\2x_1 + 4x_2 + 5x_3 + 9x_4 &= 26.\end{aligned}$$

w5.3) Balance the following chemical equation. You do this by solving the linear system that arises when you equate the number of each type of atom (*K, Fe, C, N, S, H, O*) on each side of the equation. This will give you 7 equations in 7 unknowns, but there will be a free parameter in the solution. Choose a value of the free parameter so that the numbers of each molecule are integers with no common factor.



For example, the equation that arises if you equate the number of oxygen atoms is

$$4x_2 + x_3 = 4x_4 + 4x_5 + 4x_6 + x_7.$$

If you're sick of reducing matrices by hand (and you know you could do it), use technology to compute the reduced row echelon form of the matrix that arises in this problem.