Math 2250-4
Week 2 concepts and homework, due January 18.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

There is also a Maple/software assignment posted this week, and we will run introductory Maple and Matlab sessions in LCB 115. See our homework page for details.

1.3:
3, 6, 9: geometric meaning of how slope fields relate to graphs of solutions to DE's and to IVP's
***11**, 12, 13,**14**: understanding the existence-uniqueness theorem for solutions to IVP's*

w2.1) Consider the differential equation from 13, 14 above, but a new initial value problem:

$$\frac{dy}{dx} = y^{\frac{1}{3}}$$
$$y(2) = 0.$$

w2.1a) Since the slope function is a continuous function of x, y for all $(x, y) \in \mathbb{R}^2$ we know there is at least one solution to the initial value problem above on some open interval I containing $x = 2$. Explain why the uniqueness theorem does not guarantee that this solution is unique, no matter how short the interval I is.
w2.1b) Find the general separation of variable solutions to

$$\frac{dy}{dx} = y^{\frac{1}{3}}$$

(These will have a constant "C" in them.) There is also a constant "singular solution", that separation of variables misses. What is this solution and why does separation of variable miss it?
w2.1c) Use your work in **w2.1b** to find two different solutions to the initial value problem with $y(2) = 0$ that you considered in **w2.1a**, so that each solution is actually defined on the entire real line $I = \mathbb{R} = (-\infty, \infty)$, and so that these solutions do not agree on any open interval containing $x = 2$. Thus the conditions guaranteeing uniqueness failed, and in fact there was not a unique solution on any subinterval.

w2.2a) Consider a function whose graph $y = y(x)$ has the property that the normal line at every point (x, y) on the graph passes through the point $(0, 1)$. Suppose this graph also contains the point $(x, y) = (1, 0)$. (This is a special case of a non-underlined homework problem from last week, namely 1.1.29.) Show that the function $y(x)$ satisfies the initial value problem

$$\frac{dy}{dx} = \frac{-x}{y - 1} \quad .$$
$$y(1) = 0.$$

w2.2b) Using the version of the existence-uniqueness theorem on page 28 of the text (and in our class notes), determine whether there is a unique solution to this IVP on some open interval containing the point $x = 1$.
w2.2c) Use "dfield" to plot the slope field and the graph of the solution to the initial value problem above, within the rectangle $-2 < x < 2$, $-2 < y < 2$. Recall that you can find the applet "dfield" with google. Print out a copy of this picture. Explain why the solution does not appear to exist for the entire interval $-2 < x < 2$.
w2.2d) Notice that the differential above is separable (section 1.4). Solve the initial value problem for

$y = y(x)$ and explain why the dfield picture in part (c) is consistent with your solution function. Does your solution make sense in terms of the original description of the solution's graph?

1.4:
*2, 3,**4**,9,13,19,**21**: solving DE's and IVP's for separable differential equations*
*41, **42**, 45, **46**, 49, **60**: modeling and solving problems with first order separable DEs.* Note: *The correct answer to 42 is 4.25 billion years, not the 1.25 billion years that appears in the back of the book.*

1.5:
*1, 7, **8,** 13, **24,** solving linear DE's and IVP's.*
*34,**36, 38, 41:** modeling and solving problems with first order linear DE's.*