Math 2250-4

Week 13 concepts and homework, due Friday April 12.

Recall that all problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

Carry over problems from last week, for section 6.2: diagonalizability: recognizing matrices for which there is an eigenbasis, and those for which there is not one. Writing the eigenbasis equation as AP = PD where P is the matrix of eigenvector columns, and D is the diagonal matrix of eigenvalues.
6.2: 3, 9, 21, 23.

In problem w12.4 last week, you found eigenvalues and eigenvectors (or bases for the eigenspaces if the eigenspaces were more than one-dimensional), for five different matrices. Use that data (or use technology or the solutions to last week's homewor to recover it), in order to complete the two postponed problems w12.5, w12.6 below:

w12.5)
a) Which of the matrices in w12.4 are diagonalizable and which are not?
b) For the diagonalizable matrices *A*, *C* above, verify the diagonalizing identity $P^{-1} A\, P = \mathrm{D}$ by checking the equivalent and easier to check equation $A\, P = P\, \mathrm{D}$.
c) Explain what happens to the diagonal matrix D of eigenvalues for a diagonalizable matrix, if you change the order of the eigenvector columns in *P*, in the identity $A\, P = P\, \mathrm{D}$.

w12.6)
Compute A^{10} by hand, for the matrix *A* in w12.4. Hint: use the identity $A = P\, \mathrm{D}\, P^{-1}$. (You can check your answer with technology, but don't have to hand that part in.)

7.1: modeling coupled mass-spring systems or multi-component input-output systems with systems of differential equations;converting single differential equations or systems of differential equations into equivalent first order systems of differential equations by introducing functions for the intermediate derivatives; comparing solutions to these equivalent systems.
7.1: 1, 3, **2**, 5, **8**, 11,12, 21, **24**, **26**.

w13.1) This is related to problems11, 21 ideas above.
a) Consider the IVP for the first order system of differential equations for $x(t),\, v(t)$:

$$x'(t) = v$$
$$v'(t) = -9\,x$$
$$x(0) = x_0.$$
$$v(0) = v_0 .$$

Find the equivalent second order differential equation initial value problem for the function $x(t)$.
b) Use Chapter 5 techniques to solve the second order IVP for $x(t)$in part **a**.
c) Use your result from **b** to deduce the solutions $[x(t),\, v(t)]^T$ for the IVP in **a.**

d) Use your solution formulas for $x(t)$, $v(t)$ from **c** along with algebra and trig identities to verify that the parametric solution curves $[x(t), v(t)]^T$ to the IVP in **a** lie on ellipses in the $x - v$ plane, satisfying implicit equations for ellipses given by

$$x^2 + \frac{v^2}{9} = C^2, \text{ where } C^2 = x_0^2 + \frac{v_0^2}{9}.$$

e) Reproduce the result of **d** without using the solution formulas, but instead by showing that whenever $x'(t) = v$ and $v'(t) = -9\,x$ then it must be true also that

$$\frac{d}{dt}\left(x(t)^2 + \frac{v(t)^2}{9}\right) \equiv 0,$$

so that $x(t)^2 + \frac{v(t)^2}{9}$ must be constant for any solution trajectory. Hint: use the chain rule to compute the time derivative above, then use the DE's in **a** to show that the terms cancel out.

f) What does your result in **e** have to do with conservation of energy for the undamped harmonic oscillator with mass $m = 1$ and spring constant $k = 9$? Hint: Recall that the total energy for a moving undamped mass-spring configuration with mass m and spring constant k is

$$TE = KE + PE = \frac{1}{2}m\,v^2 + \frac{1}{2}k\,x^2$$

g) Use pplane to create a picture of the tangent field for the first order system of differential equations in this problem, as well as selected solution trajectories. This picture should be consistent with your work above. Print out a screen shot to hand in.

7.2) recognizing homogeneous and non-homogeneous linear systems of first order differential equations; writing these systems in vector-matrix form; statement of existence and uniqueness for IVP's in first order systems of DE's and it's consequences for the dimension of the solution space to the first order system, and for the general solution to the non-homogeneous system. Using the Wronskian to check for bases.
7.2: 1, 9, **12**, 13, **16**, **25**.

w13.2) This is a continuation of **16**, **25**
a) Use the eigenvalue-eigenvector method of section 7.3 to generate the basis for the general solution that the text told you in **16**.
b) Use pplane to draw the phase portrait for this first order system along with the parametric curve of the solution $[x(t), y(t)]^T$ to the initial value problem in **25**. Print out a screen shot of your work to hand in.

7.3) the eigenvalue-eigenvector method for finding the solution space to homogeneous constant coefficient first order systems of differential equations: real and complex eigenvalues.
7.3: 3, 13, 29, 31, **34**, 36.

w13.3) Use the eigenvalue-eigenvector method to solve the initial value problem for the two coupled tanks on pages 1 and 3 of our Friday April 5 class notes, that we discuss on Monday. Your solution should be consistent with the picture on page 3 of those notes.

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

w13.4) Use the eigenvalue-eigenvector method (with complex eigenvalues) to solve the first order system initial value problem which is equivalent to the second order differential equation IVP on page 6 of the Friday April 5 notes, that we discuss on Monday. This is the reverse procedure from Monday, when we use the solutions from the equivalent second order DE IVP to deduce the solution to the first order system IVP. Of course, your answer here should agree with our work there.

$$\begin{bmatrix} x'(t) \\ v'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix}$$

$$\begin{bmatrix} x(0) \\ v(0) \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$