

Math 2250-4  
Week 12 concepts and homework, due April 5.

Recall that all problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

*10.4: using convolution integrals to find inverse Laplace of  $F(s)G(s)$ ; using the differentiation theorem to find the Laplace transform of  $t \cdot f(t)$ ; using these (and other) techniques to solve linear differential equations.*

10.4: 2, 3, 9, 15, 26, 29, 30, 37, 36

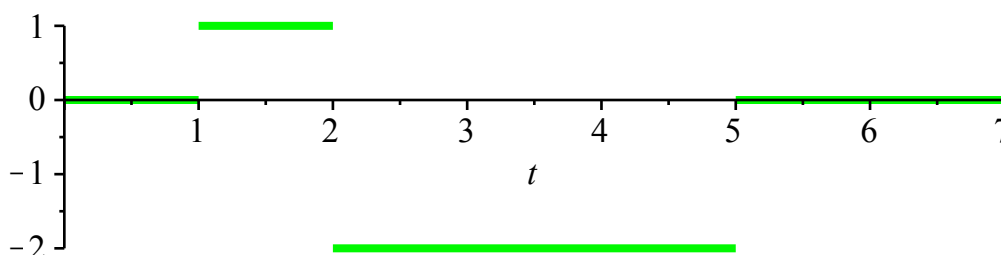
**w12.1a)** Compute the convolution  $f * g(t)$  for  $f(t) = (1 + t)$  and  $g(t) = t^2$  and use the Laplace transform table to show that  $\mathcal{L}\{f * g(t)\}(s) = F(s)G(s)$  in this case.

**b)** Repeat part a for the example  $f(t) = t, g(t) = \sin(kt)$ .

*10.5: using the unit step function to turn forcing on and off; Laplace transform of non-sinusoidal periodic functions*

10.5: 3, 7, 11, 13, 25, 31, 26, 34

**w12.2)** In a week 11 homework problem you solved the following Laplace transform problem using the definition of Laplace transform. This week, write  $f(t)$  as a linear combination of unit step functions in order to find (the same) Laplace transform. Here's the graph of  $f$ , which is zero for  $t \geq 5$ :



*EP 7.6 impulse functions as limits of differences of scaled unit step functions.*

**w12.3)** Solve this initial value problem and then sketch the solution. Explain what happens. The forcing term is a sum of instantaneous unit impulses, as discussed in EP7.6 and Monday's class notes.

$$\begin{aligned}x''(t) + 0.25x(t) &= \delta(t) + 2\delta(t - 4\pi) - 3\delta(t - 8\pi). \\x(0) &= 0 \\x'(0) &= 0\end{aligned}$$

*6.1: finding eigenvalues and eigenvectors (or a basis for the eigenspace if the eigenspace is more than one-dimensional).*

6.1: 7, 17, 19, 25

*6.2: diagonalizability: recognizing matrices for which there is an eigenbasis, and those for which there is not one. Writing the eigenbasis equation as  $AP = PD$  where  $P$  is the matrix of eigenvector columns, and  $D$  is the diagonal matrix of eigenvalues.*

6.2: 3, 9, 21, 23.

**w12.4)** Find eigenvalues and eigenvectors (or a basis for the eigenspace if the eigenspace is more than one-dimensional), for the following matrices. You can check your work with technology, but you don't have to hand in the technology check.

$$\mathbf{a)} \quad A := \begin{bmatrix} -4 & -3 \\ 6 & 5 \end{bmatrix}$$

$$\mathbf{b)} \quad B := \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\mathbf{c)} \quad C := \begin{bmatrix} 1 & 9 & 3 \\ -2 & -6 & 0 \\ 2 & 6 & 0 \end{bmatrix}$$

$$\mathbf{d)} \quad E := \begin{bmatrix} -1 & 6 & 6 \\ 0 & -3 & -2 \\ 0 & 4 & 3 \end{bmatrix}$$

$$\mathbf{e)} \quad F := \begin{bmatrix} 5 & 3 & -9 \\ -4 & -5 & 4 \\ 4 & 2 & -7 \end{bmatrix}.$$

**w12.5)** POSTPONED UNTIL APRIL 12

**a)** Which of the matrices in 12.4 are diagonalizable and which are not?

**b)** For the diagonalizable matrices  $A$ ,  $C$  above, verify the diagonalizing identity  $P^{-1} A P = D$  by checking the equivalent and easier to check equation  $A P = P D$ .

**c)** Explain what happens to the diagonal matrix  $D$  of eigenvalues for a diagonalizable matrix, if you change the order of the eigenvector columns in  $P$ , in the identity  $A P = P D$ .

**w12.6)** POSTPONED UNTIL APRIL 12

Compute  $A^{10}$  by hand, for the matrix  $A$  in 12.4. Hint: use the identity  $A = P D P^{-1}$ . (You can check your answer with technology, but don't have to hand that part in.)