Math 2250-4
Week 10 concepts and homework, due March 22

Recall that all listed problems are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems.

5.5: finding particular solutions using the method of undetermined coefficients; using variation of parameters; using the general solution $y = y_p + y_H$ *to solve associated initial value problems.*
5.5: 2, 3, 10, 21, 27, 29, **32**, 34.

w10.1) Consider the 3^{nd} order differential operator for $y(x)$:
$$L(y) := y''' + 5\,y'' + 11\,y' + 15\,y\,.$$

a) Find the solution space to the homogeneous differential equation $L(y) = 0$. Hint: first find an integer root of the characteristic polynomial, then do long division.
b) Use the method of undetermined coefficients to find a particular solution to $L(y) = x$.
c) Use the method of undetermined coefficients to find a particular solution to $L(y) = e^{x}$.
d) Use the method of undetermined coefficients to find a particular solution to $L(y) = e^{-3x}$. Hint: this will involve a lot of product rule differentiation if you just plug in the trial solution. An alternate shortcut you might consider would be to factor L so that $(D + 3\,I)$ is one of the factors. We do an example like this on Monday March 18, using Friday March 8 notes.
e) Use your earlier work and linearity (superposition) to find the general solution to
$$L(y) = 15\,x + 16\,e^{x} - 8\,e^{-3x}.$$
f) Hand in a technology check (e.g. Maple or Wolfram alpha) for your answer to **e**. If you need to, explain why the technology check actually agrees with your answer to **e**.

5.5: non-standard right-hand sides; variation of parameters.
5.5: **43**, **45**, 47, **52**, 57, **58** .

5.6: *solving forced oscillation problems; understanding beating and resonance in undamped problems, steady periodic and transient solutions in damped problems, and practical resonance in slightly damped problems; mass-spring applications; finding natural frequencies in more general (undamped) conservative systems via conservation of energy equations.*
5.6: 3, 5, 7, 8, 11, 13, 17, 18, **20**, 21, **22** .

In the following three problems we consider the forced oscillator differential equation for $x(t)$:

$$m\,x'' + c\,x' + k\,x = F_0 \cos(\omega\,t)\,.$$

We will take $m = 1\ kg$, $k = .25\,\frac{N}{m}$, $F_0 = 4\ N$, and will vary c, ω .

w10.2) Consider the configuration above, in the undamped case of the configuration above, i.e. $c = 0$. In particular consider the initial value problem

$$x'' + 0.25\,x = 4\cos(\omega\,t)$$
$$x(0) = 0$$
$$x'(0) = 0\,.$$

a) What is the natural angular frequence ω_0 (for the unforced problem) in this differential equation? Solve the IVP in case $\omega \neq \omega_0$: Use the method of undetermined coefficients for the particular solution x_P and use $x = x_P + x_H$ to solve the IVP. Check your answer with technology.

b) Write down the special case of the solution in **a** when $\omega = 0.4$. Compute the period of this solution, which is a superposition of two cosine functions. Use technology to graph one period of the solution. What phenomenon is somewhat exhibited by this solution?

c) Solve the IVP when $\omega = \omega_0$. Use the method of undetermined coefficients to find a particular solution, and then use $x = x_P + x_H$ to solve the IVP. Check your answer with technology. Graph the solution on the interval $0 \le t \le 50$ seconds. What phenomenon is exhibited by this solution?

w10.3) Now consider a damped version of the forced oscillator equation:

$$x'' + x' + 0.25\,x = 4\cos(t)$$

a) Use the method of undetermined coefficients to find a particular solution.

b) Use your work from **a** and the homogeneous solution to write down the general solution to this differential equation.

c) Identify the "steady periodic" and "transient" parts of the general solution above.

d) Use technology to solve the initial value problem for the DE above, with $x(0) = 0, x'(0) = 0$.

e) Use technology to create a graph which shows the steady periodic solution and the IVP solution in one display. Choose a $t-$ interval for which the convergence of the IVP solution to the steady periodic solution is well-illustrated.

w10.4) Investigate practical resonance for the slightly damped forced oscillator

$$x'' + .2\,x' + 0.25\,x = 4\cos(\omega\,t)\ :$$

Use the formula (21) on page 359 of the text (which we also discuss in class) to create a plot of the amplitude C of the steady periodic solution $x_{sp}(t)$ as a function of the driving angular frequency ω.

Explain why the steady periodic amplitude peaks at a value near $\omega = 0.5$. Use calculus to find the exact location of the maximum amplitude.

EP3.7: understanding unforced and forced oscillation problems for RLC circuits:

EP3.7: 11, 12, 17, **20** . In **20** use technology to solve the IVP, identify the steady periodic solution (which you don't need to convert into amplitude-phase form), and to create the graphs.