

Name _____

Student I.D. _____

Math 2250-4
Exam 2
March 28, 2013

Please show all work for full credit. This exam is closed book and closed note. You may use a scientific calculator, but not one which is capable of graphing or of solving differential or linear algebra equations. **In order to receive full or partial credit on any problem, you must show all of your work and justify your conclusions.** There are 100 points possible. The point values for each problem are indicated in the right-hand margin. **Good Luck!**

	Score	POSSIBLE
1	_____	20
2	_____	25
3	_____	25
4	_____	20
5	_____	10
TOTAL	_____	100

1a) Are $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 9 \\ 1 \end{bmatrix}$ a basis for \mathbb{R}^3 ? If they are, explain why. If they aren't a basis for \mathbb{R}^3 , determine what sort of subspace of \mathbb{R}^3 they do span, and find a basis for that subspace.

(10 points)

1b) Are $y_1(x) = e^{2x}, y_2(x) = 1$ a basis for the solution space of the differential equation

$$y''''(x) - 2y''(x) = 0?$$

If so, explain why. If they aren't a basis, explain why not, and find a correct basis for the solution space.
(10 points)

2) A focus in this course is a careful analysis of the mathematics and physical phenomena exhibited in forced and unforced mechanical (or electrical) oscillation problems. Using the mass-spring model, we've studied the differential equation for functions $x(t)$ solving

$$m x'' + c x' + kx = F_0 \cos(\omega t)$$

with $m, k, \omega > 0$; $c, F_0 \geq 0$.

2a) Explain what each of the letters m, k, c, F_0, ω represent in this model. Also give their units in the *mks* system.

(10 points)

Explain what values of c, F_0, ω lead to the phenomena listed below. What form will the key parts of the solutions $x(t)$ have in those cases, in order that the physical phenomena be present? (We're not expecting the precise formulas for these parts of the solutions, just what their forms will be.)

2b) underdamped oscillations

(3 points)

2c) pure resonance

(4 points)

2d) beating

(4 points)

2e) practical resonance.

(4 points)

3a) Use Chapter 5 techniques to solve the initial value problem

$$x''(t) + 6x'(t) + 9x(t) = 18 \cos(3t)$$

$$x(0) = 0$$

$$x'(0) = 2$$

(20 points)

Hint: first use the method of undetermined coefficients to find a particular solution. (This is the "easy" Case 1 of undetermined coefficients, because there is damping!) Even if you can't find the correct undetermined coefficients, you can still proceed through the rest of the problem to find the IVP solution, in terms of those undetermined coefficients. You will get partial credit for doing so.

3b) Write the steady periodic part of your solution to part (a) in amplitude phase form,

$$x_{sp}(t) = C \cos(3t - \alpha).$$

(5 points)

4) Use the Laplace transform technique to re-solve the same IVP as in problem (3):

$$x''(t) + 6x'(t) + 9x(t) = 18 \cos(3t)$$

$$x(0) = 0$$

$$x'(0) = 2$$

If you can't find the correct partial fraction coefficients for $X(s)$, you can still use the Laplace transform table to deduce what the solution $x(t)$ is, in terms of these unknown coefficients. You will get partial credit for doing so.

(20 points)

5) Use the Laplace transform table to find the inverse Laplace transform of

$$F(s) = \frac{4s + 10}{(s^2 + 2s + 10)} + \frac{5}{(s^2 + 4)^2}.$$

(10 points)

Table of Laplace Transforms

This table summarizes the general properties of Laplace transforms and the Laplace transforms of particular functions derived in Chapter 10.

Function	Transform	Function	Transform
$f(t)$	$F(s)$	e^{at}	$\frac{1}{s-a}$
$af(t) + bg(t)$	$aF(s) + bG(s)$	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$f'(t)$	$sF(s) - f(0)$	$\cos kt$	$\frac{s}{s^2 + k^2}$
$f''(t)$	$s^2 F(s) - sf(0) - f'(0)$	$\sin kt$	$\frac{k}{s^2 + k^2}$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$	$\cosh kt$	$\frac{s}{s^2 - k^2}$
$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$	$\sinh kt$	$\frac{k}{s^2 - k^2}$
$e^{at} f(t)$	$F(s-a)$	$e^{at} \cos kt$	$\frac{s-a}{(s-a)^2 + k^2}$
$u(t-a)f(t-a)$	$e^{-as} F(s)$	$e^{at} \sin kt$	$\frac{k}{(s-a)^2 + k^2}$
$\int_0^t f(\tau)g(t-\tau) d\tau$	$F(s)G(s)$	$\frac{1}{2k^3}(\sin kt - kt \cos kt)$	$\frac{1}{(s^2 + k^2)^2}$
$tf(t)$	$-F'(s)$	$\frac{t}{2k} \sin kt$	$\frac{s}{(s^2 + k^2)^2}$
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	$\frac{1}{2k}(\sin kt + kt \cos kt)$	$\frac{s^2}{(s^2 + k^2)^2}$
$\frac{f(t)}{t}$	$\int_s^\infty F(\sigma) d\sigma$	$u(t-a)$	$\frac{e^{-as}}{s}$
$f(t)$, period p	$\frac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t) dt$	$\delta(t-a)$	e^{-as}
1	$\frac{1}{s}$	$(-1)^{\lfloor t/a \rfloor}$ (square wave)	$\frac{1}{s} \tanh \frac{as}{2}$
t	$\frac{1}{s^2}$	$\left\lfloor \frac{t}{a} \right\rfloor$ (staircase)	$\frac{e^{-as}}{s(1-e^{-as})}$
t^n	$\frac{n!}{s^{n+1}}$		
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$		
t^a	$\frac{\Gamma(a+1)}{s^{a+1}}$		