Math 2250-4
Wednesday April 24
Course review

Final exam: Friday April 26 10:30 a.m -12:30 p.m. (I will let you work until 12:45). We will use our lecture room LCB 219 (here). I will also try to find a nearby overflow room. As usual the exam is closed book and closed note, and the only sort of calculator that is allowed is a simple scientific one. The algebra and math on the exam should all be doable by hand.

Review of previous final exam: Thursday April 25, 10:00 a.m.-noon in AEB 350.

The final exam will be comprehensive, but weighted to more recent material. Rough percentage ranges per chapter are below – these percentage ranges add up to more than 100% because many topics span several chapters.

Chapters

- 1-2: 10-20% first order DEs
- 3-4: 20-30% matrix algebra and vector spaces
- 5, EP3.7: 15-30% linear differential equations and applications
- 6.1-6.2: 15-30% eigenvalues and eigenvectors, including complex case
- 7.1-7.4: 20-40% linear systems of differential equations and applications
- 9.1-9.4: 15-25% non-linear autonomous systems of DEs and applications
- 10.4-10.5, EP 7.6: 15-25% Laplace transform

On the next page is a more detailed list of the topics we've investigated this semester. They are more inter-related than you may have realized at the time, so let's discuss the connections. Then we'll work an extended problem that lets us highlight these connections and review perhaps 70% of the key ideas in this course.

1-2: first order DEs

- slope fields
- phase diagrams for autonomous DEs
  - equilibrium solutions
  - stability
- existence-uniqueness thm for IVPs
- methods:
  - separable
  - linear
- applications
  - populations
  - velocity-acceleration models
  - input-output models

3-4 matrix algebra and vector spaces

- linear systems and matrices
- reduced row echelon form
- matrix and vector algebra
  - manipulating and solving matrix-vector equations for unknown vectors or matrices.
- matrix inverses
- determinants
- vector space concepts
  - vector spaces and subspaces
  - linear combinations
  - linear dependence/independence
  - span
  - basis and dimension
  - linear transformations
    - aka superposition
    - fundamental theorem for solution space to L(y)=f when L is linear

5 Linear differential equations

- IVP existence and uniqueness
- Linear DEs
  - Homogeneous solution space, its dimension, and why
  - superposition, $\underline{x}(t)= \underline{x}_P+ \underline{x}_H$
- Constant coefficient linear DEs
  - $\underline{x}_H$ via characteristic polynomial
    - Euler's formula, complex roots
  - $\underline{x}_P$ via undetermined coefficients
  - solving IVPs
- applications:
  - mechanical configurations
    - unforced: undamped and damped
      - cos and sin addition angle formulas and amplitude-phase form
    - forced undamped: beating, resonance
    - forced damped: $\underline{\mathbf{x}}_{sp}+ \underline{\mathbf{x}}_{tr}$, practical resonance
    - RLC circuits
    - Using conservation of total energy (=KE+PE) to derive equations of motion, especially for mass-spring and pendulum

6.1-6.1 eigenvalues, eigenvectors (eigenspaces), diagonalizable matrices…including complex eigendata.

7.1-7.4 linear systems of DEs

- first order systems of DEs and tangent vector fields.
- existence-uniqueness thm for first order IVPs
- superposition, $\underline{\mathbf{x}}= \underline{\mathbf{x}}_P+ \underline{\mathbf{x}}_H$
- dimension of solution space for $\underline{\mathbf{x}}_H$.
- conversion of DE IVPs or systems to first order system IVPs.
- Constant coefficient systems and methods:
  - $\underline{\mathbf{x}}'(t)= A\underline{\mathbf{x}}$
  - $\underline{\mathbf{x}}'(t)= A\underline{\mathbf{x}}+\underline{\mathbf{f}}(t)$
  - $\underline{\mathbf{x}}''(t)= A\underline{\mathbf{x}}$ (from conservative systems)
  - $\underline{\mathbf{x}}''(t)= A\underline{\mathbf{x}}+\underline{\mathbf{f}}(t)$
- applications: phase portrait interpretation of unforced oscillation problems; input-output modeling; force and unforced mass-spring systems.

9.1-9.4 non-linear systems of differential equations

- autonomous systems of first order DEs
  - equilibrium solutions
  - stability
  - phase portraits
  - linearization near equilibria, stability analysis, further classification and qualitative sketching.
  - Applications to interacting populations and non-linear mechanical configurations.

10.1-10.5, EP7.6: Laplace transform

- definition, for direct computation
- using table for Laplace and inverse Laplace transforms … including for topics after the second midterm.
- Solving linear DE (or system of DE) IVPs with Laplace transform.

We can illustrate many ideas in this course, and how they are tied together by studying the following two differential equations in as many ways as we can think of.

$x''(t) + 5\, x'(t) + 4\, x(t) = 0$

$x''(t) + 5\, x'(t) + 4\, x(t) = 3 \cos(2t)$