Name________________________

Student I.D.__________________

Math 2250-4
Quiz 9 SOLUTIONS
March 23, 2012

1) Consider the following mechanical oscillation differential equations. In each case answer the following questions:

(i) What is the undetermined coefficients "guess" for the particular solution $x_P(t)$? (Do NOT try to find the precise particular solution, just its form.)
(ii) What physical phenomenon will be exhibited by the general solutions to this differential equation?

Each problem is worth three points, with one free point.

1a) $x''(t) + 10\,x(t) = \cos(3\,t)$.

(i) $x_P(t) = A\cos(3\,t)$ because there are only even derivatives. You could also use $x_P(t) = A\cos(3\,t) + B\sin(3\,t)$, in which case you would end up with $B = 0$.

(ii) Because the driving (angular) frequency $\omega = 3$ is close to the natural (angular) frequency $\omega_0 = \sqrt{10}$, solutions are likely to experience <u>beating</u> (for almost all choices of initial conditions).

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> with(DEtools) :
  dsolve({x''(t) + 10·x(t) = cos(3·t), x(0) = 0, x'(0) = 0});
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$$x(t) = -\cos\left(\sqrt{10}\,t\right) + \cos(3\,t) \qquad \textbf{(1)}$$

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> with(plots) :
  plot(-cos(sqrt(10.0)·t) + cos(3·t), t = 0..100, color = black, title = `beating`);
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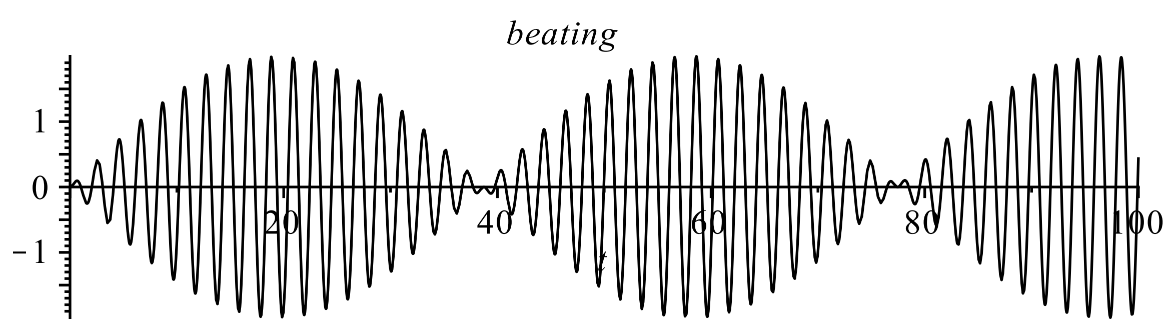


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1b) $x''(t) + 9\,x(t) = \cos(3\,t)$

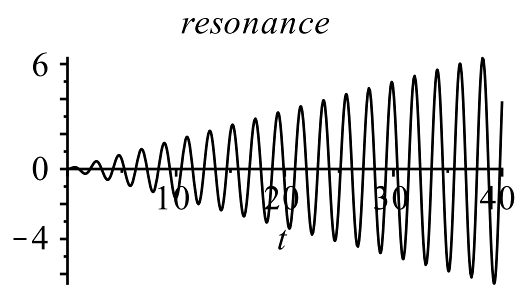
(i) $x_P(t) = t\,(A\cos(3\,t) + B\sin(3\,t))$ (Because $span\{\cos(3\,t), \sin(3\,t)\}$ is associated to the complex solutions $e^{3\,i\,t}\ e^{-3\,i\,t}$ to the homogeneous DE, and the characteristic polynomial $p(r) = (r - 3\,i)^1(r + 3\,i)^1$ we multiple the guess we made in 1a by t^1 .

(ii) Resonance!

> $dsolve(\{x''(t) + 9 \cdot x(t) = \cos(3 \cdot t), x(0) = 0, x'(0) = 0\});$

$$x(t) = \frac{1}{6} \sin(3\, t)\, t \tag{2}$$

> $plot\left(\frac{1}{6} \cdot \sin(3 \cdot t) \cdot t, t = 0..40, color = black, title = `resonance`\right);$



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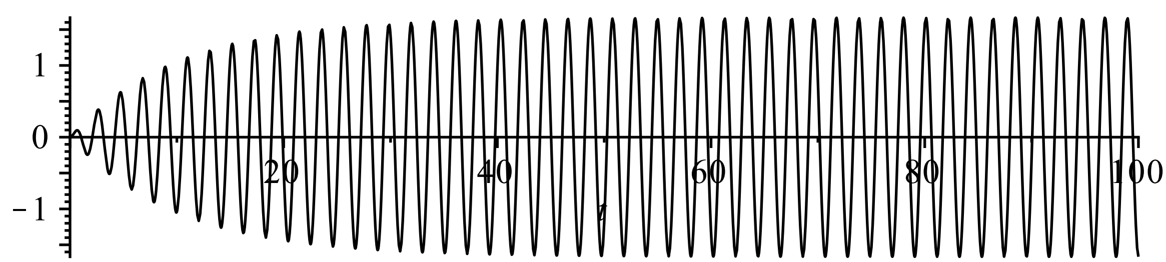
1c) $x''(t) + 0.2 \cdot x'(t) + 9\, x(t) = \cos(3\, t)$

(i) $x_p(t) = A \cos(3\, t) + B \sin(3\, t)$ - because $\pm 3\, i$ are NOT roots of the characteristic poly for the homogeneous DE.
(ii) Because the damping coefficient .2 is small, and since we are forcing with a frequency that would be the natural frequence with no damping, we expect a significant response relative to the amplitude of the input forcing function, in the steady periodic part of the solutions (which this x_p will be). This is practical resonance. (We are not forcing at precisely the frequence that will induce the largest amplitude response, but we are very close to it.)

> $dsolve(\{x''(t) + .2 \cdot x'(t) + 9 \cdot x(t) = \cos(3 \cdot t), x(0) = 0, x'(0) = 0\});$

$$x(t) = -\frac{50}{899} e^{-\frac{1}{10} t} \sin\left(\frac{1}{10} \sqrt{899}\, t\right) \sqrt{899} + \frac{5}{3} \sin(3\, t) \tag{3}$$

> $plot\left(-\frac{50}{899} e^{-\frac{1}{10} t} \sin\left(\frac{1}{10} \sqrt{899}\, t\right) \sqrt{899} + \frac{5}{3} \sin(3\, t), t = 0..100, color = black\right);$



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