Name

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Math 2250-4 Quiz 7 March 2, 2012

1) Consider the differential equation for y(x)

$$y'' + 7 y' + 12 y = 0$$

1a) Find the general solution to this differential equation.

(5 points)

We try for a basis of exponential function solutions....if $y = e^{rx}$ then $L(y) = y'' + 7y' + 12y = e^{rx}(r^2 + 7r + 12) = e^{rx}(r + 4)(r + 3)$ so $y_1(x) = e^{-4x}$, $y_2(x) = e^{-3x}$ solve the DE. Thus the general solution is

$$y_H(x) = c_1 e^{-4x} + c_2 e^{-3x}$$

How to check in Maple: with (DEtools): dsolve(y''(x) + 7 · y'(x) + 12 · y(x) = 0, y(x)); y(x) = _C1 e^{-4x} + _C2 e^{-3x} (1)

1b) What is the dimension of the solution space above?

(1 point)

The dimension is 2, since there are two functions in a basis for the solution space. (In general we know that the solution space to nth order homogeneous linear differential equations is n-dimensional.)

2) Now consider the non-homogeneous differential equation

$$y'' + 7y' + 12y = -24.$$

Notice that a particular solution to this differential equation is the constant function $y_P = -2$. Use this particular solution and your work in problem (1) in order to solve the initial value problem

$$y'' + 7y' + 12y = -24$$

 $y(0) = -3$
 $y'(0) = 1$.

(4 points)

Note - there was a typo on the exam so that the DE in the IVP did not match the DE above it. Grading will be generous for people who tried working with the different DE and ran out of time. (Most of you seem to have assumed it was a typo - but please ask next time this happens, so that I can clarify what was meant.)

$$y = y_P + y_H = -2 + c_1 e^{-4x} + c_2 e^{-3x}$$

$$y'(x) = -4 c_1 e^{-4x} - 3 c_2 e^{-3x}.$$

At x = 0 we match the initial data to get

$$-2 + c_1 + c_2 = -3$$

$$-4 c_1 - 3 c_2 = 1$$

i.e.

$$\begin{array}{c} c_1 + c_2 = -1 \\ -4 c_1 - 3 c_2 = 1 \\ c_2 \end{array} \begin{bmatrix} c_1 \\ -4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -4 & -3 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = -\frac{1}{-1} \begin{bmatrix} -3 & -1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

so

$$y(x) = -2 + 2e^{-4x} - 3e^{-3x}.$$

Maple check:

with (DEtools):
dsolve(
$$\{y''(x) + 7 \cdot y'(x) + 12 \cdot y(x) = -24, y(0) = -3, y'(0) = 1\}$$
);
 $y(x) = 2 e^{-4x} - 3 e^{-3x} - 2$
(2)

For the (luckily only several) people who didn't assume the DE in the IVP was a typo, here's how your work would have turned out with the DE that was actually written there:

$$y'' + 5y' + 6y = -24$$

$$y(0) = -3$$

$$y'(0) = 1$$

$$y_{H}: \text{ when you try } y = e^{rx} \text{ you get } L(e^{rx}) = e^{rx}(r^{2} + 5r + 6) = e^{rx}(r + 3)(r + 2) \text{ so}$$

$$y_{H}(x) = c_{1}e^{-3x} + c_{2}e^{-2x}$$

By inspection, we may take $y_{1} = -4$ since $L(-4) = 0 + 0 + 6(-4) = -24$. Thus

By inspection, we may take $y_p = -4$, since L(-4) = 0 + 0 + 6(-4) = -24. Thus

$$y = y_P + y_H = -4 + c_1 e^{-3x} + c_2 e^{-2x}$$
.

To solve the IVP, compute

$$y(0) = -4 + c_1 + c_2 = -3$$

y'(0) = -3 c_1 - 2 c_2 = 1,

i.e.

$$c_{1} + c_{2} = 1$$

$$-3 c_{1} - 2 c_{2} = 1.$$

$$\begin{bmatrix} c_{1} \\ c_{2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3 & -2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{1} \begin{bmatrix} -2 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

$$y(x) = -4 - 3e^{-3x} + 4e^{-2x}.$$

so