Name________________________
Student I.D.___________________

Math 2250−4
Quiz 6
February 24, 2012

1a) What does it mean for a vector $\underline{\mathbf{v}}$ to be a <u>linear combination</u> of the vectors $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2, \ldots \underline{\mathbf{v}}_n$?

(1 point)

A linear combination of $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2, \ldots \underline{\mathbf{v}}_n$ *is any vector which is a sum of scalar multiples of* $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2, \ldots \underline{\mathbf{v}}_n$ *, i.e. any* $\underline{\mathbf{v}}$ *which can be expressed as*

$$\underline{\mathbf{v}} = c_1\underline{\mathbf{v}}_1 + c_2\underline{\mathbf{v}}_2 + \ldots c_n\underline{\mathbf{v}}_n .$$

1b) What is the <u>span</u> of a collection of vectors $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2, \ldots \underline{\mathbf{v}}_n$?

(1 points)

The span of $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2, \ldots \underline{\mathbf{v}}_n$ *is the set of all possible linear combinations.*

1c) What does it mean for vectors $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2, \ldots \underline{\mathbf{v}}_n$ to be <u>linearly independent</u>?

(1 point)

$\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2, \ldots \underline{\mathbf{v}}_n$ *are* <u>*linearly independent*</u> *if the only way* $c_1\underline{\mathbf{v}}_1 + c_2\underline{\mathbf{v}}_2 + \ldots c_n\underline{\mathbf{v}}_n = \underline{\mathbf{0}}$ *is for* $c_1 = c_2 = \ldots c_n = 0$.

2) Find a basis for the span of the following five vectors in $\mathbb{R}^4$. Explain your reasoning. You may find the reduced row echelon form computation below useful. The five vectors are

$$\underline{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \ \underline{\mathbf{v}}_2 = \begin{bmatrix} -2 \\ -4 \\ -2 \\ -4 \end{bmatrix}, \ \underline{\mathbf{v}}_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \ \underline{\mathbf{v}}_4 = \begin{bmatrix} -2 \\ -3 \\ 0 \\ -4 \end{bmatrix}, \ \underline{\mathbf{v}}_5 = \begin{bmatrix} 1 \\ 0 \\ -4 \\ 2 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & -2 & 0 & -2 & 1 \\ 2 & -4 & 1 & -3 & 0 \\ 1 & -2 & 2 & 0 & -4 \\ 2 & -4 & 0 & -4 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(7 points)

As we discussed on 2/22 in Exercise 5 of the notes of 2/21 notes, you delete dependent vectors from the original set of 5, until you're left with an independent spanning set. This was also the content of homework exercise w7.6.

Because column dependencies are preserved when we reduce the matrix, we read off that $\underline{\mathbf{v}}_2 = -2\,\underline{\mathbf{v}}_1$ *and* $\underline{\mathbf{v}}_4 = -2\,\underline{\mathbf{v}}_1 + \underline{\mathbf{v}}_3$ *, whereas* $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_3, \underline{\mathbf{v}}_5$ *are linearly independent. Thus*

$$span\{\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_2, \underline{\mathbf{v}}_3, \underline{\mathbf{v}}_4, \underline{\mathbf{v}}_5\} = span\{\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_3, \underline{\mathbf{v}}_5\}$$

and $\underline{\mathbf{v}}_1, \underline{\mathbf{v}}_3, \underline{\mathbf{v}}_5$ *are a basis for the span of all 5 columns.*

<u>*Note:*</u> *A lot of people tried instead to a basis for the solution space* $A\,\underline{\mathbf{x}} = \underline{\mathbf{0}}$ *. That's not what was asked for.*