Name\_\_\_\_\_

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## Math 2250-4 Quiz 5 **Solutions** February 10, 2012

1) Consider the following system of equations

2x + 2y + 2z = 4x + 2y - z = 1 $-2\dot{y} + 4z = 3$ 

Exhibit the augmented matrix corresponding to this system, compute its reduced row echelon form, and find the solution set to the system.

(7 points)

	0 -2 4 3
$\frac{R_1}{2} \to R_1$	
	1 2 -1 1
	0 -2 4 3
$-R_1 + R_2 \rightarrow R_2$	
	$\begin{bmatrix} 1 & 1 & 1 & 2 \end{bmatrix}$
	0 1 -2 -1
	0 -2 4 3
$2R_2 + R_3 \rightarrow R_3$	
	$\begin{bmatrix} 1 & 1 & 1 & 2 \end{bmatrix}$
	0 1 -2 -1
	0 0 0 1

already at this stage we see that the bottom row corresponds to the equation 0 x + 0 y + 0 z = 1 which has no solutions so our original system is inconsistent and the solution set is empty. (no solutions).

$$R_{3} + R_{2} \rightarrow R_{2}, -2 R_{3} + R_{1} \rightarrow R_{1}$$

$$\begin{bmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 1 & -2 & | & 0 \\ 0 & 0 & 0 & | & 1 \end{bmatrix}$$

$$-R_{2} + R_{1} \rightarrow R_{1}$$

**^**2  $\mathbf{n}_1$ 

> with(LinearAlgebra) :
 ReducedRowEchelonForm(Matrix(3, 4, [2, 2, 2, 4, 1, 2, -1, 1, 0, -2, 4, 3]));
 #check answer for fun

2) Consider the matrix equation

$$\begin{bmatrix} 1 & -2 & 3 & 4 \\ 5 & -2 & 7 & 3 \\ 3 & 2 & 1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Without finding the solution set explicitly, explain which of the following three outcomes are possible for the solution set, just based on the number of equations, the number of unknowns, and the right hand side: (a) no solutions; (b) exactly one solution; (c) infinitely many solutions.

(3 points) Because this is a homogeneous matrix equation the zero vector  $\mathbf{x} = \mathbf{0}$  (i.e.  $x_1 = x_2 = x_3 = x_4 = 0$ ) is a

solution to the system. Thus outcome (a) cannot happen. Because there are four columns and only three rows in *A* there will be at least one column in its reduced row echelon form without a leading one, so there will be a free parameter when we backsolve, i.e. infinitely many solutions. (As it turns out, there are only two non-zero rows in the reduced row echelon form so there are two free parameters in the general solution.) Thus, only outcome (c) is possible.

for fun, here's the reduced row echelon form: