Name_________________________
Student I.D.____________________

Math 2250−4
Quiz 4 Solutions
February 3, 2012

1) Suppose that an object moves vertically, subject only to the acceleration of gravity $g = 32\ \frac{ft}{s^2}$ and a drag force proportional to the object's velocity. Choose the positive y direction to be up and write $y'(t) = v(t)$ for the velocity. For particular values of the object's mass and the drag coefficient, the differential equation

$$\frac{dv}{dt} = -32 - 0.5 \cdot v$$

governs the object's velocity $v(t)$.

1a) Construct a phase diagram and determine $\lim_{t \to \infty} v(t)$ for all solutions to this differential equation. What is the term for this limiting velocity?

(3 points)

$$v'(t) = -0.5(v + 64)$$

so the equilibrium solution for velocity is $v = -64\ \frac{ft}{s}$ *, and* $v'(t) > 0$ *for* $v < -64$ *and* $v'(t) < 0$ *for* $v > -64$ *. Thus the phase diagram looks like*

$$\rightarrow \rightarrow \rightarrow \rightarrow -64 \leftarrow \leftarrow \leftarrow \leftarrow \leftarrow.$$

and $\lim_{t \to \infty} v(t) = -64$ *for any solution to the DE. This limiting value is called the* <u>*terminal velocity*</u>*.*

1b) Solve the following initial value problem for the differential equation above,

$$\frac{dv}{dt} = -32 - 0.5 \cdot v$$

$$v(0) = 40 .$$

(5 points)

Note that the object is initially thrown upwards.

$$v'(t) + .5\, v(t) = -32$$

$$\frac{d}{dt}\left(e^{.5\,t}\, v(t)\right) = e^{.5\,t}(-32)$$

$$e^{.5\,t} v = \int -32\, e^{.5\,t} dt = -64\, e^{.5\,t} + C$$

$$v = -64 + C\, e^{-.5\,t}.$$

$v(0) = 40 = -64 + C \Rightarrow C = 104 \Rightarrow$

$$v = -64 + 104\, e^{-.5\,t}$$

1c) Find a formula for the height $y(t)$ of the object, assuming $y(0) = 0$.

(2 points)

$$y(t) = y_0 + \int_0^t v(s)\, ds = 0 + \int_0^t -64 + 104\, e^{-.5\,s}\, ds$$

$$= \left[-64\,s - 208\,e^{-.5\,s} \right]_0^t = -64\,t + 208\left(1 - e^{-.5\,t}\right).$$

There's not time to ask on this quiz, but could you figure out the maximum height that the object attains?
Set $v(t) = 0$ and solve for t. Plug this value of t into the formula for $y(t)$ to find the maximum height.