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Student I.D.__________________

Math 2250-4
Quiz 12 Solutions
April 20, 2012

1) Find the general solution $\left[x_1(t), x_2(t)\right]^T$ to the homogeneous system of second order differential equations, which could result from two masses coupled together with springs:

$$x_1''(t) = -3\,x_1 + x_2$$
$$x_2''(t) = 2\,x_1 - 2\,x_2\,.$$

(8 points)

$$\begin{bmatrix} x_1''(t) \\ x_2''(t) \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\begin{vmatrix} -3-\lambda & 1 \\ 2 & -2-\lambda \end{vmatrix} = (\lambda+3)(\lambda+2) - 2 = \lambda^2 + 5\,\lambda + 4$$
$$= (\lambda+4)(\lambda+1)\,.$$

$\lambda = -1$ $\left(\omega = \sqrt{-\lambda} = 1\right)$ the homogeneous system is

$$\left[\begin{array}{cc|c} -2 & 1 & 0 \\ 2 & -1 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} -2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \underline{\boldsymbol{v}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

$\lambda = -4$ $(\omega = 2)$ the homogeneous system is

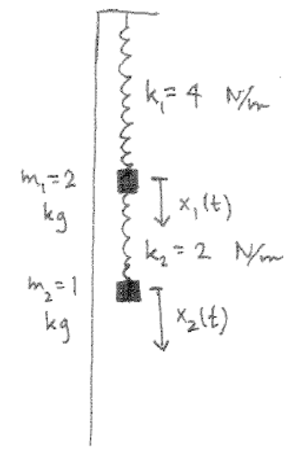
$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & 2 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \underline{\boldsymbol{v}} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

So,

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \left(c_1\cos(t) + c_2\sin(t)\right)\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \left(c_3\cos(2\,t) + c_4\sin(2\,t)\right)\begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

2) Use Newton's second law to derive the system of differential equations above, from the configuration below. $x_1(t), x_2(t)$ are the displacements from equilibrium of two masses. The mass values and spring constants are as shown.

(2 points)



$$2\,x_1''(t) = -4\,x_1 + 2(x_2 - x_1) = -6\,x_1 + 2\,x_2$$
$$1\,x_1''(t) = -2(x_2 - x_1) = 2\,x_1 - 2\,x_2\ .$$

This is equivalent to the system in problem 1 (divide the first DE by 2).