Name_____

Student I.D._____

Math 2250-4
Quiz 11 Solutions
April 13, 2012

1a) Use the methods we've been discussing to find the general solution to the system of differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

(8 points)

$$\begin{vmatrix} 0-\lambda & 1 \\ -6 & -5-\lambda \end{vmatrix} = \lambda(\lambda+5)+6 = \lambda^2+5\lambda+6 = (\lambda+3)(\lambda+2)$$

$\lambda = -3$ *the homogeneous system is*

$$\left[\begin{array}{cc|c} 3 & 1 & 0 \\ -6 & -2 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 3 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \underline{v} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$\lambda = -2$ *the homogeneous system is*

$$\left[\begin{array}{cc|c} 2 & 1 & 0 \\ -6 & -3 & 0 \end{array}\right] \to \left[\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow \underline{v} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

Thus the general solution is

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 e^{-2t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} + c_2 e^{-3t} \begin{bmatrix} 1 \\ -3 \end{bmatrix}.$$

1b) For any solution $[x(t), y(t)]^T$ to the first order system of DEs above, what is the second order differential equation satisfied by $x(t)$?

(2 points)

The first order system reads

$$x'(t) = y$$
$$y'(t) = -6x - 5y.$$

Thus $x'' = y' = -6x - 5y = -6x - 5x'$. *We can also write this DE in the usual form for an unforced undamped mechanical or electrical system*

$$x'' + 5x' + 6x = 0.$$