Math 2250-4
Tues Mar 27

homework session 3:30-4:30 p.m. today in JFB 103
go over last fall's midterm 5-6:30 p.m. today, also in JFB 103

Wednesday will be a review day for the exam on Thursday. Remember, we start 5 minutes early and end 5 minutes late, so 2250-5 students will take the exam from 10:40-11:40 and 2250-6 students will take it from 11:45-12:45.

10.1-10.3 Laplace transform, and application to DE IVPs, including Chapter 5.
Today we'll continue to fill in the Laplace transform table (on page 2), and to use the table entries to solve linear differential equations. One focus today will be to review partial fractions, since the table entries are set up precisely to show the inverse Laplace transforms of the components of partial fraction decompositions.

Exercise 1) Check why this table entry is true - notice that it generalizes how the Laplace transforms of $\cos(k t)$, $\sin(k t)$ are related to those of $e^{a t}\cos(k t)$, $e^{a t}\sin(k t)$:

$e^{a t} f(t)$	$F(s-a)$

- Next, use Tuesday's notes to complete the concepts and discussion related to Exercises 6-8 in those notes.

- Then, here's a complement to Exercise 8 on Tuesday's notes (in which we considered the resonant case):

Exercise 2) Use Laplace transforms to solve the general undamped forced oscillation problem, when $\omega \neq \omega_0$:

$$x''(t) + \omega_0^2 x(t) = F_0 \sin(\omega t)$$
$$x(0) = x_0$$
$$x'(0) = v_0$$

when $\omega \neq \omega_0$.

$f(t)$, with $\lvert f(t)\rvert \le Ce^{Mt}$	$F(s) := \int_0^{\infty} f(t)e^{-st}\,dt$ for $s > M$	↓ verified
$c_1 f_1(t) + c_2 f_2(t)$	$c_1F_1(s) + c_2F_2(s)$	☐
1 t t^2 $t^n,\ n \in \mathbb{N}$	$\frac{1}{s}$ $(s > 0)$ $\frac{1}{s^2}$ $\frac{2}{s^3}$ $\frac{n!}{s^{n+1}}$	☐
$e^{\alpha t}$	$\frac{1}{s-\alpha}$ $(s > \Re(a))$	☐
$\cos(k\,t)$ $\sin(k\,t)$ $\cosh(k\,t)$ $\sinh(k\,t)$ $e^{a\,t}\cos(k\,t)$ $e^{a\,t}\sin(k\,t)$ $e^{a\,t} f(t)$	$\frac{s}{s^2+k^2}$ $(s > 0)$ $\frac{k}{s^2+k^2}$ $(s > 0)$ $\frac{s}{s^2-k^2}$ $(s > k)$ $\frac{k}{s^2-k^2}$ $(s > k)$ $\frac{(s-a)}{(s-a)^2+k^2}$ $(s > a)$ $\frac{k}{(s-a)^2+k^2}$ $(s > a)$ $F(s-a)$	☐ ☐ ☐ ☐ ☐ ☐
$f'(t)$ $f''(t)$ $f^{(n)}(t),\ n \in \mathbb{N}$ $\int_0^t f(\tau)\,d\tau$	$s\,F(s) - f(0)$ $s^2F(s) - s\,f(0) - f'(0)$ $s^n\,F(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$ $\frac{F(s)}{s}$	☐ ☐ ☐ ☐
$t\,f(t)$ $t^2\,f(t)$ $t^n\,f(t),\ n \in \mathbb{Z}$ $\frac{f(t)}{t}$	$-F'(s)$ $F''(s)$ $(-1)^n\,F^{(n)}(s)$ $\int_s^{\infty} F(\sigma)\,d\sigma$	
$t\,\cos(k\,t)$ $\frac{1}{2\,k}t\,\sin(k\,t)$	$\frac{s^2-k^2}{(s^2+k^2)^2}$ $\frac{s}{(s^2+k^2)^2}$	

$\frac{1}{2k^3}(\sin(k t) - k t \cos(k t))$ $t e^{a t}$ $t^n e^{a t},\ n \in \mathbb{Z}$	$\frac{1}{(s^2 + k^2)^2}$ $\frac{1}{(s - a)^2}$ $\frac{n!}{(s - a)^{n+1}}$	
more after the midterm!		

Laplace transform table

Exercise 3) Check these table entries two ways, using work you've already done.

$t e^{a t}$ $t^n e^{a t}$	$\frac{1}{(s - a)^2}$ $\frac{n!}{(s - a)^{n+1}}$

Exercise 4) Solve the following IVP. Use this example to recall the general partial fractions algorithm.

$$x''(t) + 4x(t) = 8 t e^{2t}$$
$$x(0) = 0$$
$$x'(0) = 1$$

Exercise 5a) What is the form of the partial fractions decomposition for

$$X(s) = \frac{-356 + 45\,s - 100\,s^2 - 4\,s^5 - 9\,s^4 + 39\,s^3 + s^6}{(s-3)^3\,\left((s+1)^2+4\right)\,\left(s^2+4\right)}\,.$$

5b) Have Maple compute the precise partial fractions decomposition.

5c) What is $x(t) = \mathcal{L}^{-1}\{X(s)\}(t)$?

5d) Have Maple compute the inverse Laplace transform directly.

> $X := s \rightarrow \dfrac{-356 + 45\cdot s - 100\cdot s^2 - 4\cdot s^5 - 9\cdot s^4 + 39\cdot s^3 + s^6}{(s-3)^3\cdot\left((s+1)^2+4\right)\cdot\left(s^2+4\right)^2};$

> $convert(X(s), parfrac, s);$

> $with(inttrans);$

> $invlaplace(X(s), s, t);$

>