Math 2250-4

Week 4 concepts and homework, due February 3.

Recall that problems which are not underlined are good for seeing if you can work with the underlying concepts; that the underlined problems are to be handed in; and that the Friday quiz will be drawn from all of these concepts and from these or related problems. You will also have a Maple/Matlab project due on Monday February 6.

2.3: improved velocity-acceleration models:
2, *9,* ***10, 12****: constant, or constant plus linear drag forcing*
13, 14, 17*, 18: quadratic drag*
24, 25*: escape velocity*

2.4-2.6: *numerical methods for approximating solutions to first order initial value problems. Your second Maple/Matlab project addresses these sections too.*
2.4: ***5****: Euler's method*
2.5: ***5****: improved Euler*
2.6: ***5****: Runge-Kutta*

w4.1) Runge-Kutta is based on Simpson's rule for numerical integration. Simpson's rule is based on the fact that for a subinterval of length $2 \cdot h$, which by translation we may assume is the interval $-h \le x \le h$, the parabola $y = p(x)$ which passes through the points $(-h, y_{-1})$, $(0, y_0)$, (h, y_1) has integral

$$\int_{-h}^{h} p(x)\, dx = \frac{2\,h}{6} \cdot (y_{-1} + 4\,y_0 + y_1)\,. \qquad (1)$$

If we write the quadratic interpolant function $p(x)$ whose graph is this parabola as

$$p(x) = a + b\,x + c\,x^2$$

with unknown parameters a, b, c then since we want $p(0) = y_0$ we solve $y_0 = a + 0 + 0$ to deduce that $a = y_0$.

w4.1a) Use the requirement that the graph of $p(x)$ is also to pass through the other two points, $(-h, y_{-1})$, (h, y_1) to express b, c in terms of h, y_{-1}, y_0, y_1.

w4.1b) Compute $\int_{-h}^{h} p(x)\, dx$ for these values of a, b, c and verify equation (1) above.